KD EDUCATION ACADEMY [9582701166] Street no. 21 A-1 block Bengali colony sant nagar burari delhi -110084

Time : 6 hour

STD 9 Maths

kd sir 90+ Question ch-2 polynomial

Total Marks : 310

*** Choose the right answer from the given options. [1 Marks Each] [66]**

1. The product $(a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2)$ is equal to:

(A) $a^6 + b^6$ (B) $a^6 - b^6$ (C) $a^3 - b^3$ (D) $a^3 + b^3$

Ans. :

b. $a^6 - b^6$

Solution:

$(a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2)$

$= (a^2 - b^2)(a^2 + b^2 - ab)(a^2 + b^2 - ab)$

$= (a^2 - b^2)\left\{(a^2 + b^2)^2 - (ab)^2\right\}$

$= (a^2 - b^2)\{a^4 + b^4 + 2a^2b^2 - a^2b^2\}$

$= (a^2 - b^2)\{a^4 + b^4 + a^2b^2\}$

$= \{a^6 + a^2b^4 + a^4b^2 - b^2a^4 - b^6 - b^4a^2\}$

$= a^6 - b^6$

Hence, correct option is (b).

2. If $x + y + z = 9$ and $xy + zx = 23$, then the value of $x^3 + y^3 + z^3 - 3xyz$ is:

(A) 144 (B) 108 (C) 209 (D) 180

Ans. :

b. 108

Solution:

Given: $x + y + z = 9$ and $xy + zx = 23$

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x + y + z)[(x + y + z)^2 - 2xy - 2yz - 2zx - xy - yz - zx]$

$= (x + y + z)[(x + y + z)^2 - 3xy - 3yz - 3zx]$

$= (x + y + z)[(x + y + z)^2 - 3(xy + yz + zx)]$

$= 9 \times [81 - 69]$

$= 9 \times 12$

$= 108$

3. If $(m^2 - 3)x^2 + 3mx + 3m + 1 = 0$ has roots which are reciprocal of each other, then the value of m equals

(A) 4 (B) 1 (C) 2 (D) None of these.

Ans. :

a. 4

Solution:

If the root are reciprocal then the product of the roots of the equation equals to 1.

$(m^2 - 3)x^2 + 3mx + 3m + 1 = 0$

Product of the roots $= \frac{c}{a}$

$\frac{3m+1}{m^2-3} = 1$

or $m^2 - 3m - 4 = 0$

or $m^2 - 4m + m - 4 = 0$

or $(m + 1)(m - 4) = 0$

$m = 4$ or $m = -1$

$\therefore 4$

4. The value of $\frac{0.75\times0.75\times0.75+0.25\times0.25\times0.25}{0.75\times0.75-0.75\times0.25+0.25\times0.25}$ is:

(A) -1 (B) 2 (C) 1 (D) 0

Ans. :

c. 1

Solution:

$\frac{0.75\times0.75\times0.75+0.25\times0.25\times0.25}{0.75\times0.75-0.75\times0.25+0.25\times0.25}$

$= \frac{(0.75)^3+(0.25)^3}{(0.75)^2-0.75\times0.25+(0.25)^2}$

$= \frac{(0.75+0.25[(0.75)^2-0.75\times0.25+(0.25)^2]}{(0.75)^2-0.75\times0.25+(0.25)^2}$

$= 0.75 + 0.25$

$= 1$

5. If $x + \frac{1}{x} = 3$, then $x^6 + \frac{1}{x^6} =$

(A) 927 (B) 414 (C) 364 (D) 322

Ans. :

d. 322

Solution:

$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$x + \frac{1}{x} = 3$ (given)

$\Rightarrow x^2 + \frac{1}{x^2} = (3)^2 - 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 7 \ldots (1)$

Cubing both side of equation (1). we have

$\left(x^2 + \frac{1}{x^2}\right)^3 = (7)^3$

$\Rightarrow (x^2)^3 + \left(\frac{1}{x^2}\right)^3 + 3(x^2)\frac{1}{x^2}\left(x^2 + \frac{1}{x^2}\right) = 7^3$

$\Rightarrow x^6 + \frac{1}{x^6} + 3(7) = 7^3$

$\Rightarrow x^6 + \frac{1}{x^6} = 343 - 21$

$\Rightarrow x^6 + \frac{1}{x^6} = 322$

Hence, correct option is (d).

6. If $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^2$, then k =

(A) 1 (B) 2 (C) 4 (D) 8

Ans. :

d. 8

Solution:

Let x + y = A and x - y = B

Now, $(A - B)^3 = A^3 - B^3 - 3AB(A - B)$

$\Rightarrow [(x + y) - (x - y)]^3 = (x + y)^3 - (x - y)^3 - 3(x + y)(x - y)[(x + y) - (x - y)]$

$= (x + y)^3 - (x - y)^3 - 3(x^2 - y^2)(2y)$

$= (x + y)^3 - (x - y)^3 - 6y(x^2 - y^2)$

But, $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^3$

$\Rightarrow [(x + y) - (x - y)]^3 = (2y)^3 = k8y^3$

$\Rightarrow (2y)^3 = ky^3$

$\Rightarrow 8y^3 = ky^3$

$\Rightarrow k = 8$

Hence, correct option is (d).

7. If x + 2 and x - 1 are the factor of $x^3 + 10x^2 + mx + n$, then the values of m and n are respectively.

(A) 5 and -3 (B) 7 and -18 (C) 23 and -19 (D) 17 and -8

Ans. :

b. 7 and -18

Solution:

It is given (x + 2) and (x - 1) are the factors of the polynomial $f(x)= x^3 + 10x^2 + mx + n$

i.e., f(-2) = 0 and f(1) = 0

New,

$f(-2) = (-2)^3 + 10(-2)^2 + m(-2) + n = 0$

$-8 + 40 - 2m + n = 0$

$\Rightarrow -2m + n = -32$

$\Rightarrow 2m - n = 32$ ....(i)

$f(1) = (1)^3 + 10(1)^2 + m(1) + n = 0$

$1 + 10 + m + n = 0$

$m + n = -11$ .....(ii)

Solving equation (i) and (ii) we get,

m = 7 and n = -18

8. If $49a^2 - b = \left(7a + \frac{1}{2}\right)\left(7a - \frac{1}{2}\right)$, then the value of b is:

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

Ans. :

b. $\frac{1}{4}$

Solution:

$\left(7a+\frac{1}{2}\right)\left(7a-\frac{1}{2}\right)=(7a)^2-\left(\frac{1}{2}\right)^2$

[by using identity $(a + b)(a - b) = a^2 - b^2$]

$\Rightarrow \left(7a+\frac{1}{2}\right)\left(7a-\frac{1}{2}\right)=49a^2-\frac{1}{4}$

$\Rightarrow 49a^2-b=49a^2-\frac{1}{4}$

$\Rightarrow b=\frac{1}{4}$

Hence, correct option is (b).

9. The value of $\frac{(0.87)^3+(0.13)^3}{(0.87)^2-(0.87\times0.13)+(0.13)^2}$ is:

(A) 0 (B) 0.13 (C) 0.87 (D) 1

Ans. :

d. 1

Solution:

$\frac{(0.87)^3+(0.13)^3}{(0.87)^2-(0.87\times0.13)+(0.13)^2}$

$=\frac{(0.87+0.13)[(0.87)^2-(0.87\times0.13)+(0.13)^2]}{(0.87)^2-(0.87\times0.13)+(0.13)^2}$

$=0.87+0.13$

$=1$

10. $(a - b)^3 + (b - c)^3 + (c - a)^3 =$

(A) $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ (B) $(a - b)(b - c)(c - a)$ (C) $3(a - b)(b - c)(c - a)$ (D) None of these.

Ans. :

c. $3(a - b)(b - c)(c - a)$

Solution:

Let

$a - b = A$

$b - c = B$

$c - a = C$

Now $(A + B + C)^3 = A^3 + B^3 + C^3 + 3(A + B)(B + C)(C + A)$

$\Rightarrow A^3 + B^3 + C^3 = (A + B + C)^3 - 3(A + B)(B + C)(C + A)$

Now putting values of A, B and C. we get

$(a-b)^3+(b-c)^3+(c-a)^3$

$=(\cancel{a}-\cancel{b}+\cancel{b}-\cancel{c}+\cancel{c}-\cancel{a})^3$

$-3(a-\cancel{b}+\cancel{b}-c)(b-\cancel{c}+\cancel{c}-a)(c-\cancel{a}+\cancel{a}-b)$

$\Rightarrow (a - b)^3 + (b - c)^3 + (c - a)^3 = 0 - 3(a - c)(b - a)(c - b)$

$\Rightarrow (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$

Hence, correct option is (c).

11. If a + b + c = 0, then $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} =$

(A) 0 (B) 1 (C) -1 (D) 3

Ans. :

d. 3

Solution:

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

If a + b + c = 0, then

$a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$ ...(1)

Now, consider $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$

Multiplying dividing by a. b. and c in $\frac{a^2}{bc}$. $\frac{b^2}{ca}$ and $\frac{c^2}{ab}$ respectively. we get

$\frac{a^3}{abc} + \frac{b^3}{bca} + \frac{c^3}{cab}$

$= \frac{a^3+b^3+c^3}{abc}$

$= \frac{3abc}{abc}$ ....[From (1)]

$= 3$

Hence, correct option is (d).

12. If x + y = 8 and xy = 15, than $x^2 + y^2$

(A) 32 (B) 1 (C) 34 (D) 36

Ans. :

c. 34

Solution:

$x^2 + y^2 = (x + y)^2 - 2xy$

$\Rightarrow x^2 + y^2 = (8)^2 - 2 \times 15$

$\Rightarrow x^2 + y^2 = 64 - 30$

$\Rightarrow x^2 + y^2 = 34$

13. Write the correct answer in the following:

The value of $249^2 - 248^2$ is.

(A) 1^2 (B) 477 (C) 487 (D) 497

Ans. :

d. 497

Solution:

$(249)^2 - (248)^2 = (249 + 248)(249 - 248)$ $[(a)^2 - (b)^2 = (a + b)(a - b)]$

= (497)(1) = 497

14. If p(x) = (x - 1) (x + 1), then the value of p(2) + p(1) - p(0) is:

(A) 2 (B) 4 (C) 1 (D) 3

Ans. :

b. 4

Solution:

Given: p(x) = (x - 1) (x + 1), then

p(2) + p(1) - p(0)

= (2 - 1) (2 + 1) + (1 - 1) (1 + 1) - (0 - 1) (0 + 1)

= 1 × 3 + 0 × 2 -(-1) × 1

= 3 + 0 + 1

= 4

15. If $\frac{a}{b}+\frac{b}{a}=-1$ then (a^3 - b^3) = ?

(A) -3 (B) -2 (C) -1 (D) 0

Ans. :

d. 0

Solution:

$\frac{a}{b}+\frac{b}{a}=-1$

$\Rightarrow \frac{a^2+b^2}{ab}=-1$

⇒ a^2 + b^2 = -ab

⇒ a^2 + b^2 + ab = 0

Thus, we have:

(a^3 - b^3) = (a - b)(a^2 + b^2 + ab)

= (a - b) × 0

= 0

16. If x + y + z = 9 and xy + yz + zx = 23, the value of (x^3 + y^3 + z^3 - 3xyz) = ?

(A) 108 (B) 207 (C) 669 (D) 729

Ans. :

a. 108

Solution:

x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)

= (x + y + z)[$(x + y + z)^2$ - 3(xy + yz + zx)]

= 9 × (81 - 3 × 23)

= 9 × 12

= 108

17. If $x+\frac{1}{x}=3$, then $x^6+\frac{1}{x^6}=$

(A) 927 (B) 322 (C) 414 (D) 364

Ans. :

b. 322

Solution:

On cubing we get.

$$\left(x+\frac{1}{x}\right)^3=x^3+\left(\frac{1}{x^3}\right)+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)$$

$$\Rightarrow 27 = x^3 + \left(\frac{1}{x^3}\right) + 3 \times 3$$

$$\Rightarrow x^3 + \left(\frac{1}{x^3}\right) = 27 - 9$$

$$\Rightarrow x^3 + \left(\frac{1}{x^3}\right) = 18$$

Now, $\left(x^3 + \frac{1}{x^3}\right)^2 = x^6 + \left(\frac{1}{x^6}\right) + 2 \times x^3 \times \frac{1}{x^3}$

$$\Rightarrow 18^2 = x^6 + \left(\frac{1}{x^6}\right) + 2$$

$$x^6 + \left(\frac{1}{x^6}\right) = 324 - 2 = 322$$

18. If $(3x - 1)^7 = a_7x^7 + a_6x^6 + a_5x^5 +$ ___ $+ a_1x + a_0$, then $a_7 + a_6 + a_5 +$ ____ $+ a_1 + a_0 =$

(A) 0 (B) 128 (C) 1 (D) 64

Ans. :

b. 128

Solution:

Given that,

$(3x - 1)^7 = a_7x^2 + a_5x^5 +$ ___ $+ a_1x + a_0$

Putting x = 1

We get

$(3 \times 1 - 1)^7 = a_6(1)^5 + a_5(1)^5$ ____ $+ a_1(1) + a_0$

$\Rightarrow a_6 + a_5 +$ ____ $+ a_1 + a_0 = 2^7 = 128$

19. If $\frac{a}{b} + \frac{b}{a} = 1$, then $a^3 + b^3 =$

(A) 1 (B) -1 (C) 0 (D) $\frac{1}{2}$

Ans. :

c. 0

Solution:

Here, $\frac{a}{b} + \frac{b}{a} = 1$

$$\Rightarrow \frac{a^2+b^2}{ab} = 1$$

$$\Rightarrow a^2 + b^2 = ab$$

$$\Rightarrow a^2 + b^2 - ab = 0$$

Using, $a^2 + b^2 = (a + b)(a^2 + b^2 - ab)$

$$= (a + b)(0)$$

$$= 0$$

20. The value of $\frac{(0.013)^3+(0.007)^3}{(0.013)^2-0.013\times0.007+(0.007)^2}$, is:

(A) 0.0091 (B) 0.006 (C) 0.00185 (D) 0.02

Ans. :

d. 0.02

Solution:

Assume a = 0.013 and v = 0.007.

Than the given expression can be rewritten as $\frac{a^3+b^3}{a^2-ab+b^2}$

Recall the formula for sum of two cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Using the above formula, the expression becomes $\frac{(a+b)(a^2-ab+b^2)}{(a^2-ab+b^2)}$

Note that both a and b are positive. So, neither $a^3 + b^3$ nor any factor of it can be zero.

Therefore we can cancel the term $(a^2 - ab + b^2)$ from both numerator and denominator. then the expression becomes

$$\frac{(a+b)(a^2-ab+b^2)}{a^2-ab+b^2} = a + b$$
$$= 0.013 + 0.007$$
$$= 0.02$$

21. The product $(x^2 - 1)(x^4 + x^2 + 1)$ is equal to:

(A) $x^8 - 1$ (B) $x^8 + 1$ (C) $x^6 - 1$ (D) $x^6 + 1$

Ans. :

c. $x^6 - 1$

Solution:

Given expression is $(x^2 - 1)(x^4 + x^2 + 1)$

Let x^2 = A and 1 = B

Then, we have

$(A - B)(A^2 + AB + B^2)$

$= A^3 - B^3$

$= (X^2)^3 - (1)^3$

$= X^6 - 1$

Hence, correct option is (c).

22. $\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3} =$

(A) $3(a + b)(b + c)(c + a)$ (B) $3(a - b)(b - c)(c - a)$ (C) $(a - b)(b - c)(c - a)$ (D) None of these.

Ans. :

d. None of these.

Solution:

If a + b + c = 0 then, $a^3 + b^3 + c^3 = 3abc$

Now, $(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$

$\Rightarrow (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$

Again, (a - b) + (b - c) + (c - a) = a - b + b - c + c - a = 0

⇒ $(a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$

Thus, we have

$$\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$$
$$=\frac{3(a^2-b^2)(b^2-c^2)(c^2-a^2)}{3(a-b)(b-c)(c-a)}$$
$$=\frac{(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{(a-b)(b-c)(c-a)}$$
$$=(a+b)(b+c)(c+a)$$

Hence, correct option is (d).

23. If a + b + c = 0 then $\left(\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}\right)=?$

(A) 1 (B) 0 (C) -1 (D) 3

Ans. :

d. 3

Solution:

a + b + c = 0 ⇒ $a^3 + b^3 + c^3 = 3abc$

Thus, we have:

$$\left(\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}\right)=\frac{a^3+b^3+c^3}{abc}$$
$$=\frac{3abc}{abc}$$
$$=3$$

24. If the polynomial $x^3 - 6x^2 + ax + 3$ leaves a remainder 7 when divided by (x - 1), than the value of a is:

(A) 9 (B) 7 (C) 8 (D) 0

Ans. :

a. 21

Solution:

If the polynomial $x^3 - 6x^2 + ax + 3$ leaves a remainder 7 when divided by (x - 1), i.e., P(1) = 7

Now we will calculate P(1) to find the value of a

$P(1) = (1)^3 - 6(1)^2 + a(1) + 3$

⇒ 7 = 1 - 6 + a + 3

⇒ -2 + a = 7

⇒ a = 9

25. If $3x+\frac{2}{x}=7$, then $\left(9x^2-\frac{4}{x^2}\right)=$

(A) 25 (B) 35 (C) 49 (D) 30

Ans. :

b. 35

Solution:

$$\left(3x + \frac{2}{x}\right)^2 = 9x^2 + \frac{4}{x^2} + 12 \ldots (1)$$

$$\left(3x - \frac{2}{x}\right)^2 = 9x^2 + \frac{4}{x^2} - 12 \ldots (2)$$

Subtracting eq. (1) from eq. (2). we get

$$\left(3x - \frac{2}{x}\right)^2 - \left(3x + \frac{2}{x}\right)^2 = -24$$

$$\Rightarrow \left(3x - \frac{2}{x}\right)^2 = (7)^2 - 24 = 25$$

$$\Rightarrow 3x - \frac{2}{x} = 5$$

Now $\left(3x + \frac{2}{x}\right) - \left(3x - \frac{2}{x}\right) = 7 \times 5$

$$\left(9x^2 - \frac{4}{x^2}\right) = 35$$

Hence, correct option is (b).

26. The value of k for which x - 1 is a factor of $4x^3 + 3x^2 - 4x + k$, is:

(A) 3 (B) 1 (C) -2 (D) -3

Ans. :

d. -3

Solution:

Let $p(x) = 4x^3 + 3x^2 - 4x + k$

Now,

if (x - 1) is a factor of p(x), then at x = 1, p(x) = 0

So, p(1) = 0

$\Rightarrow 4(1)^3 + 3(1)^2 - 4(1) + k = 0$

$\Rightarrow 4 + 3 - 4 + k = 0$

$\Rightarrow k = -3$

27. If a + b + c = 9 and ab + bc + ca =23, then $a^3 + b^3 + c^3 - 3abc$ =

(A) 108 (B) 207 (C) 669 (D) 729

Ans. :

a. 108

Solution:

Given, a + b + c = 9

Hence, $(a + b + c)^2 = 81$

So, $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 81$

i.e. $a^2 + b^2 + c^2 + 2(ab + bc + ca) = 81$

i.e. $a^2 + b^2 + c^2 + 2(23) = 81$

i.e. $a^2 + b^2 + c^2 = 81 - 46 = 35$

Now, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= (a + b + c)[(a^2 + b^2 + c^{2)} - (ab + bc + ca)]$

= (9)[35 - 23]

$= 9 \times 12$

$= 108$

Hence, correct option is (a).

28. The value of $(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$ is:

(A) 3(a + b) (b + c) (c + a) (a - b) (b - c) (c - a)
(B) 3(a - b) (b - c) (c - a)
(C) 3(a + b) (b + c) (c + a)
(D) None of these.

Ans. :

a. 3(a + b) (b + c) (c + a) (a - b) (b - c) (c - a)

Solution:

$(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$

Here,

$a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$

Therefore,

$(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$

[Since $x^3 + y^3 + z^3 = 3xyz$ if $x + y + z = 0$]

$(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 =$

3(a + b) (b + c) (c + a) (a - b) (b - c) (c - a)

29. If $x^2 + kx - 3 = (x - 3)(x + 1)$, than the value of 'k' is:

(A) -2
(B) 2
(C) -3
(D) 3

Ans. :

a. -2

Solution:

$x^2 + kx - 3 = (x - 3)(x + 1)$,

$\Rightarrow x^2 + kx - 3 = x^2 + (-3 + 1)x + (-3) \times 1$

$\Rightarrow x^2 + kx - 3 = x^2 = 2x - 3$

On comparing the term, we get = -2

30. If $x + \frac{1}{x} = 4$, then $x^4 + \frac{1}{x^4} =$

(A) 196
(B) 194
(C) 192
(D) 190

Ans. :

b. 194

Solution:

$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$\left(x + \frac{1}{x}\right) = 4$ (given)

$\Rightarrow x^2 + \frac{1}{x^2} = (4)^2 - 2 = 16 - 2 = 14 \ldots (1)$

Squaring equation (1)

$\left(x^2 + \frac{1}{x^2}\right)^2 = (14)^2$

$$\Rightarrow (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(x^2)\frac{1}{x^2} = 196$$
$$\Rightarrow x^4 + \frac{1}{x^4} = 196 - 2$$
$$\Rightarrow x^4 + \frac{1}{x^4} = 194$$
Hence, correct option is (b).

31. If $x^4 + \frac{1}{x^4} = 194$, then $x^3 + \frac{1}{x^3} =$

(A) 76 (B) 52 (C) 64 (D) None of these

Ans. :

b. 52

Solution:

$$x^4 + \frac{1}{x^4} = 194$$
Now $\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2$
$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 194 + 2 = 196$$
$$\Rightarrow x^2 + \frac{1}{x^2} = 14 \ldots (1)$$
Now $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \left\{x^2 + \frac{1}{x^2} = 14\right\}$
$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 14 + 2 = 16$ [From (1)]
$$\Rightarrow x + \frac{1}{x} = \sqrt{16}$$
$$\Rightarrow x + \frac{1}{x} = 4 \ldots (3)$$
By identity $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
$$\Rightarrow x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$$
$$= (4)(14 - 1)$$
$$= 4 \times 13$$
$$= 52$$
Hence, correct option is (b).

32. If $x^2 + kx + 6 = (x + 2)(x + 3)$, for all x,then the value of k is:

(A) 3 (B) -1 (C) 1 (D) 5

Ans. :

d. 5

Solution:

$x^2 + kx + 6 = (x + 2)(x + 3)$,

$\Rightarrow x^2 + kx + 6 = x^2 + (2 + 3)x + 2 \times 3$

$\Rightarrow x^2 + kx + 6 = x^2 + 5x + 6$

On comparing the terms,

We get k = 5

33. The remainder when $x^{31} - 31$ is divided by x + 1 is:

(A) -32 (B) 31 (C) 30 (D) 0

Ans. :

a. -32

Solution:

x^{31} - 31

Using remainder theorem.

= $(-1)^{31}$ - 31

= -1 - 31

= -32

34. If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, then:

(A) a + b + c (B) b + c = a (C) c + a = b (D) a = b = c

Ans. :

d. a = b = c

Solution:

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

Multiplying by 2 on both the sides, we have

$2(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$

$a^2 + a^2 + b^2 + b^2 + c^2 + c^2 - 2ab - 2bc - 2ca = 0$

$(a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (a^2 + c^2 - 2ac) = 0$

$(a - b)^2 + (b - c)^2 + (a - c)^2 = 0$

$(a - b)^2 = 0, (b - c)^2 = 0, (a - c)^2 = 0$

(a - b) = 0, (b - c) = 0, (a - c) = 0

a = b, b = c, a = c

or we can say a = b = c

Hence, correct option is (d).

35. If $a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}} = 0$, than.

(A) $a^3 + b^3 + c^3 = 0$ (B) a + b + c (C) $(a + b + c)^3 = 27abc$ (D) a + b + c = 3abc

Ans. :

c. $(a + b + c)^3 = 27abc$

Solution:

$$a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}} = 0$$

$$\Rightarrow a^{\frac{1}{3}} + b^{\frac{1}{3}} = -c^{\frac{1}{3}}$$

$$\Rightarrow \left[(a^{\frac{1}{3}})(b^{\frac{1}{3}})\right]^3 = \left(-c^{\frac{1}{3}}\right)^3$$

$$\Rightarrow a + b + \left[3 \times a^{\frac{1}{3}} \times b^{\frac{1}{3}}\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\right)\right] = -c$$

$$\Rightarrow a + b + 3 \times a^{\frac{1}{3}} \times b^{\frac{1}{3}}\left(-c^{\frac{1}{3}}\right) = -c$$

$$\Rightarrow a+b+c=3\times a^{\frac{1}{3}}\times b^{\frac{1}{3}}\times c^{\frac{1}{3}}$$
$$\Rightarrow (a+b+c)^3=\left(3\times a^{\frac{1}{3}}\times b^{\frac{1}{3}}\times c^{\frac{1}{3}}\right)$$
$$\Rightarrow (a+b+c)^3=27abc$$

36. The Possible expressions for the length and breadth of the rectangle whose area is given by $4a^2 + 4a - 3$ is:

(A) (2a - 1) and (2a + 3) (B) (2a - 1) and (2a - 3) (C) (2a + 1) and (2a + 3) (D) None of these.

Ans. :

a. (2a - 1) and (2a + 3)

Solution:

$4a^2 + 4a - 3$

To find the length and breadth, we will factorize the given polynomial.

$= 4a^2 - 6a - 2a - 3$

$= 2a(a + 3) - 1(2a + 3)$

$= (2a + 3)(2a - 1)$

Therefore, the Possible expressions for the length and breadth of the rectangle whose area is given by $4a^2 + 4a - 3$ is (2a + 3) and (2a - 1).

37. If a + b + c = 9 and ab + bc + ca = 23, than $a^3 + b^3 + c^3 - 3abc$ =

(A) 729 (B) 207 (C) 669 (D) 108

Ans. :

d. 108

Solution:

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\Rightarrow (9)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow (9)^2 = a^2 + b^2 + c^2 + 2(23)$

$\Rightarrow a^2 + b^2 + c^2 = 81 - 46 = 35$

as we know that $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = 9 \times (35 - 23)$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = 108$

38. Write the correct answer in the following:

If $\frac{x}{y}+\frac{y}{x}=-1\ (x, y \neq 0)$, the value of x^3-y^3 is.

(A) 1 (B) -1 (C) 0 (D) $\frac{1}{2}$

Ans. :

c. 0

Solution:

Given, $\frac{x}{y}+\frac{y}{x}=-1$

$$\Rightarrow \frac{x^2+y^2}{xy}=-1$$

$$\Rightarrow x^2+y^2=-xy$$

$\Rightarrow x^2 + y^2 + xy = 0$

Now, $x^3 - y^3 = (x - y)(x^2 + xy + y^2) \ldots (i)$

$[a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$= (x - y) \times 0 = 0$ [From Eq. (i)]

39. If a + b + c = 0, then $\left(\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right) =$

(A) 1 (B) 3 (C) 0 (D) 2

Ans. :

b. 3

Solution:

$\left(\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right)$

$= \frac{a^3+b^3+c^3}{abc}$

Since a + b + c = 0, then $a^3 + b^3 + c^3 = 3abc$

Therefore,

$= \frac{3abc}{abc}$

$= 3$

40. If $49x^2 - k = \left(7x + \frac{1}{3}\right)\left(7x - \frac{1}{3}\right)$, than the value of 'k' is:

(A) $\frac{1}{9}$ (B) $\frac{-1}{9}$ (C) $\frac{1}{3}$ (D) $\frac{-1}{3}$

Ans. :

a. $\frac{1}{9}$

Solution:

$49x^2 - k = \left(7x + \frac{1}{3}\right)\left(7x - \frac{1}{3}\right),$

$\Rightarrow 49x^2 - k = 49x^2 - \frac{1}{9}$ [Using identity (a + b) (a - b) = $a^2 - b^2$]

On comapring $k = \frac{1}{9}$

41. If $p(x) = x^3 - x^2 + x + 1$, than the value of $\frac{p(-1)+p(1)}{2}$ is:

(A) 2 (B) 1 (C) 3 (D) 0

Ans. :

d. 0

Solution:

$p(x) = x^3 - x^2 + x + 1,$

$= \frac{p(-1)+p(1)}{2}$

$= \frac{(-1)^3-(-1)^2+(-1)+1+(1)^3-(1)^2+(1)+1}{2}$

$= \frac{-1-1-1+1+1-1+1+1}{2}$

$= \frac{0}{2}$

$= 0$

42. If $x + 2$ is a factor of $x^2 + mx + 14$, then m =

(A) 7 (B) 2 (C) 9 (D) 14

Ans. :

c. 9

Solution:

If $x + 2$ is a factor of $x^2 + mx + 14$,

then at $x = -2$,

$x^2 + mx + 14 = 0$

i.e. $(-2)^2 + m(-2) + 14 = 0$

$4 - 2m + 14 = 0$

$2m = 18$

$m = 9$

43. If $(x^{100} + 2x^{99} + k)$ is divisible By $(x + 1)$ then the value of k is:

(A) -2 (B) 1 (C) 2 (D) -3

Ans. :

b. 1

Solution:

Let: $(x^{100} + 2x^{99} + k)$

Now, $x + 1 = 0 \Rightarrow x = -1$

$\therefore p(-1) = 0$

$\Rightarrow (1)^{100} + 2 \times (-1)^{99} + k = 0$

$\Rightarrow 1 - 2 + k = 0$

$\Rightarrow -1 + k = 0$

$\Rightarrow k = 1$

44. If $x^4 + \frac{1}{x^4} = 194$, than $x^3 + \frac{1}{x}^3 =$

(A) 64 (B) 52 (C) 76 (D) None of these.

Ans. :

b. 52

Solution:

$$\left(x^4 + \frac{1}{x^4}\right) = 194$$

$$\Rightarrow (x^2)^2 + \left(\frac{1}{x^2}\right) + 2 \times x^2 \times \frac{1}{x^2} = 194 + 2 \times x^2 \times \frac{1}{x^2}$$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 196$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \sqrt{196} = 14$$

Now,

$$\Rightarrow (x^2) + \left(\frac{1}{x^2}\right) + 2 \times x \times \frac{1}{x} = 14 + 2 \times x \times \frac{1}{x}$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 16$$

$$\Rightarrow x+\frac{1}{x}=\sqrt{16}=4$$

Now, $\left(x+\frac{1}{x}\right)^{3}=(4)^{3}$

$$\Rightarrow (x)^{3}+\left(\frac{1}{x}\right)^{3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=64$$

$$\Rightarrow (x^{3})+\left(\frac{1}{x^{3}}\right)+3(4)=64$$

$$\Rightarrow (x^{3})+\left(\frac{1}{x^{3}}\right)=64-12=52$$

45. If $\left(3x+\frac{1}{2}\right)\left(3x-\frac{1}{2}\right)=9x^{2}-p$ then the value of p is:

(A) 0 (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

Ans. :

c. $\frac{1}{4}$

Solution:

$$\left(3x+\frac{1}{2}\right)\left(3x-\frac{1}{2}\right)=9x^{2}-p$$

$$9x^{2}-\frac{1}{4}\ \left(\therefore\ (a^{2}-b^{2})=(a+b)(a-b)\right)$$

$$=9x^{2}-p$$

$$\Rightarrow p=\frac{1}{4}$$

46. If $x^{2}+\frac{1}{x^{2}}=38$, then the value of $x-\frac{1}{x}$ is:

(A) 3 (B) 4 (C) 5 (D) 6

Ans. :

d. 6

Solution:

$$\left(x-\frac{1}{x}\right)^{2}=x^{2}+\frac{1}{x^{2}}-2\times x\times\frac{1}{x}$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^{2}=38-2$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^{2}=36$$

$$\Rightarrow x-\frac{1}{x}=\pm 6$$

47. If $(x + y)^3 - (x - y)^3 - 6y (x^2 - y^2) = ky^2$, then k =

(A) 1 (B) 2 (C) 8 (D) 4

Ans. :

c. 8

Solution:

We have,

$= (x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^3$

$= (x + y - x + y)^3 + 3(x + y) (x - y) (x + y - x + y) - 6y(x^2 - y^2) = ky^3$

$= 2y^3 + 6y(x^2 - y^2) - 6y(x^2 - y^2) = ky^3$

$= 8y^3 = ky^3$

$= k = 8$

48. If $3x = a + b + c$, then the value of $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c)$ is:

(A) $a + b + c$ (B) $(a - b)(b - c)(c - a)$ (C) 0 (D) None of these.

Ans. :

c. 0

Solution:

$3x = a + b + c$

$\Rightarrow a + b + c - 3x = 0$

$\Rightarrow 3x - (a + b + c) = 0$

$\Rightarrow (x - a) + (x - b) + (x - c) = 0$ ...(1)

Using identity if $a + b + c = 0$ then, $a^3 + b^3 + c^3 - 3abc = 0$

If we take $x - a = A$, $x - b = B$, $x - c = C$ in equation (1), we get

$A + B + C = 0$

$\Rightarrow A^3 + B^3 + C^3 - 3ABC = 0$

$\Rightarrow (x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c) = 0$

Hence, correct option is (c).

49. The value of $\frac{(a^2-b^2)^3(b^2-c^2)+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$ is:

(A) $3(a - b)(b - c)(c - a)$ (B) $3(a + b)(b + c)(c + a)$ (C) $3(a + b)(b + c)(c + a)(a - b)(b - c)(c - a)$ (D) None of these.

Ans. :

b. $3(a + b)(b + c)(c + a)$

Solution:

$$\frac{(a^2-b^2)^3(b^2-c^2)+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$$

$$= \frac{3(a^2-b^2)(b^2-c^2)(c^2-a^2)}{3(a-b)(b-c)(c-a)} \; [\text{Since } x^3 + y^3 + z^3 = 3xyz, \text{ if } x + y + z = 0]$$

$$= \frac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c(c)-a)}$$

$$= 3(a + b)(b + c)(c + a)$$

50. If $x^3 - 3x^2\ 3x - 7 = (x + 1)(ax^2 + bx + c)$, then $a + b + c =$

(A) 4 (B) -10 (C) 12 (D) 3

Ans. :

a. 4

Solution:

First multiply

$(x + 1)(ax^2 + bx + c)$

$= ax^3 + bx^2 + cx + ax^2 + bx + c$

$= ax^3 + bx^2 + ax^2 + cx + c$

= $ax^3 + (b + a)x^2 + (c + b)x + c$

Comparing it with

$x^3 - 3x^2 + 3x - 7$

a = 1

b + a = -3 ⇒ b + 1 + -3 ⇒ b = -4

c + b = 3 ⇒ c - 4 = 3 ⇒ c = 7

c = -7 should be 7

as if we put x = -1 in

$x^3 - 3x^2 + 3x - 7$

-1 - 3 - 3 - 7 =14 so x + 1 can not be factor so x + 1 will be factor if $x^3 - 3x^2 + 3x - 7$ is actually

$x^3 - 3x^2 + 3x + 7$

then -1 -3 -3 + 7 = 0

Hence, we can say that

a = 1

b = -1

c = 7

so, a + b + c = 4

51. Write the correct answer in the following:

If $49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$, the value of b is.

a. 0

b. $\frac{1}{\sqrt{2}}$

c. $\frac{1}{4}$

d. $\frac{1}{2}$

Ans. :

c. $\frac{1}{4}$

Solution:

$49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$

$\Rightarrow 49x^2 - b = \left(7x\right)^2 - \left(\frac{1}{2}\right)^2$

$49^2 - \frac{1}{4}[\because (a+b)(a-b) = a^2 - b^2]$

So, we get $b = \frac{1}{4}$.

52. Write the correct answer in the following:

Degree of the polynomial $4x^4 + 0x^3 + 0x^5 + 5x + 7$ is.

a. 4

b. 5

c. 3

d. 7

Ans. :

a. 4

Solution:

The height power of the variable in a polynomial is called the degree of the polynomial. In this polynomial, the term with highest power of x is $4x^4$. Highest power of x is 4, so the degree of the given polynomial is 4.

53. If $x^4 + \frac{1}{x^4} = 623$, then $x + \frac{1}{x} =$

a. 27

b. 25

c. $3\sqrt{3}$

d. $-3\sqrt{3}$

Ans. :

c. $3\sqrt{3}$

Solution:

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2.x.\frac{1}{x} = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \left\{\left(x + \frac{1}{x}\right)^2 - 2\right\}$$

Squaring both sides.

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = \left\{\left(x + \frac{1}{x}\right)^2 - 2\right\}^2$$

$$\Rightarrow x^4 + \frac{1}{x^4} + 2.x^2.\frac{1}{x} = \left\{\left(x + \frac{1}{x}\right)^2 - 2\right\}^2$$

$$\Rightarrow x^4 + \frac{1}{x^4} + 2 = \left\{\left(x + \frac{1}{x}\right)^2 - 2\right\}^2 = (623) + 2$$

$$\Rightarrow 623 + 2 = \left\{\left(x + \frac{1}{x}\right)^2 - 2\right\}^2 \left\{x^4 + \frac{1}{x^4} = 623\right\}$$

$$\Rightarrow 625 = \left\{\left(x + \frac{1}{x}\right)^2 - 2\right\}^2$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 - 2 = \sqrt{625} = 25$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 25 + 2 = 27$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = \sqrt{27}$$

$$\Rightarrow x + \frac{1}{x} = 3\sqrt{3}$$

Hence, correct option is (c).

54. The product $(a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2)$ is equal to:

a. $a^6 + b^6$

b. $a^6 - b^6$

c. $a^3 - b^3$

d. $a^3 + b^3$

Ans. :

b. $a^6 - b^6$

Solution:

$(a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2)$

$= (a^2 - b^2)(a^2 + b^2 - ab)(a^2 + b^2 - ab)$

$= (a^2 - b^2)\left\{(a^2 + b^2)^2 - (ab)^2\right\}$

$= (a^2 - b^2)\{a^4 + b^4 + 2a^2b^2 - a^2b^2\}$

$= (a^2 - b^2)\{a^4 + b^4 + a^2b^2\}$

$= \{a^6 + a^2b^4 + a^4b^2 - b^2a^4 - b^6 - b^4a^2\}$

$= a^6 - b^6$

Hence, correct option is (b).

55. If $a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}} = 0$, then:

a. $a + b + c = 0$

b. $(a + b + c)^3 = 27abc$

c. $a + b + c = 3abc$

d. $a^3 + b^3 + c^3 = 0$

Ans. :

b. $(a + b + c)^3 = 27abc$

Solution:

Let $a^{\frac{1}{3}} = A,\ b^{\frac{1}{3}} = B$ and $c^{\frac{1}{3}} = C$

Now, A + B + C = 0 (given)

If A + B + C = 0, then $A^3 + B^3 + C^3 - 3ABC = 0$

$\Rightarrow A^3 + B^3 + C^3 - 3ABC = 0$

$\Rightarrow A^3 + B^3 + C^3 = 3ABC$ ...(1)

$$\left\{\begin{array}{c} A = a^{\frac{1}{3}},\ B = b^{\frac{1}{3}},\ C = c^{\frac{1}{3}} \\ A^3 = a,\ B^3 = b,\ C^3 = c \end{array}\right\}$$

Then, equation (1) becomes

$a + b + c = 3(abc)^{\frac{1}{3}}$

Cubing both Sides of above equation, we get

$(a + b + c)^3 = 27abc$

Hence, correct option is (b).

56. If $\frac{a}{b} + \frac{b}{a} = -1$, then $a^3 - b^3 =$

a. 1

b. -1

c. $\frac{1}{2}$

d. 0

Ans. :

d. 0

Solution:

$\frac{a}{b} + \frac{b}{a} = -1$

$\Rightarrow \frac{a^2+b^2}{ab} = -1$

$\Rightarrow a^2 + b^2 + ab = 0$

Now using identity

$a^3 - b^3$

$= (a - b)(a^2 + b^2 + ab)$

$=(a - b)(0) \left(\because a^2 + b^2 + ab = 0\right)$

$= 0$

Hence, correct option is (d).

57. $\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3} =$

a. 3(a + b)(b+ c)(c + a)
b. 3(a - b)(b - c)(c - a)
c. (a - b)(b - c)(c - a)
d. None of these.

Ans. :

d. None of these.

Solution:

If a + b + c = 0 then, $a^3 + b^3 + c^3 = 3abc$

Now, $(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$

$\Rightarrow (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$

Again, (a - b) + (b - c) + (c - a) = a - b + b - c + c - a = 0

$\Rightarrow (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$

Thus, we have

$\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

$= \frac{3(a^2-b^2)(b^2-c^2)(c^2-a^2)}{3(a-b)(b-c)(c-a)}$

$= \frac{(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{(a-b)(b-c)(c-a)}$

$= (a+b)(b+c)(c+a)$

Hence, correct option is (d).

58. The expression $x^4 + 4$ can be factorized as:

a. $(x^2 + 2x + 2)(x^2 - 2x + 2)$
b. $(x^2 + 2x + 2)(x^2 + 2x - 2)$
c. $(x^2 - 2x - 2)(x^2 - 2x + 2)$
d. $(x^2 + 2)(x^2 - 2)$

Ans. :

a. $(x^2 + 2x + 2)(x^2 - 2x + 2)$

Solution:

$x^4 + 4$

$= x^4 + 4 + 4x^2 - 4x^2$

$= (x^4 + 4x^2 + 4) - 4x^2$

$= (x^2 + 2)^2 - (2x)^2$

$= (x^2 + 2 - 2x)(x^2 + 2 + 2x)$

$= (x^2 + 2x + 2)(x^2 - 2x + 2)$

Hence, correct option is (a).

59. The factors of $8a^3 + b^3 - 6ab + 1$ are:

a. $(2a + b - 1)(4a^2 + b^2 + 1 - 3ab - 2a)$
b. $(2a - b + 1)(4a^2 + b^2 - 4ab + 1 - 2a + b)$
c. $(2a + b + 1)(4a^2 + b^2 + 1 - 2ab - b - 2a)$
d. $(2a - 1 + b)(4a^2 + 1 - 4a - b - 2ab)$

Ans. :

c. $(2a + b + 1)(4a^2 + b^2 + 1 - 2ab - b - 2a)$

Solution:

We know the identity

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

So by using identity, we can write given expression as

$(2a)^3 + (b)^3 + (1)^3 - 3(2a)(b)(1)$

$= (2a + b + 1)[(2a)^2 + b^2 + 1^2 - 2a \times b - b \times 1 - 2a \times 1]$

$= (2a + b + 1)(4a^2 + b^2 + 1 - 2ab - b - 2a)$

Hence, correct option is (c).

60. If x - a is a factor of $x^3 - 3x^2a + 2a^2x + b$, then the value of b is:

a. 0
b. 2
c. 1
d. 3

Ans. :

a. 0

Solution:

Let $p(x) = x^3 - 3x^2a + 2a^2x + b$

(x - a) is a factor of p(x).

So,

p(a) = 0

$a^3 - 3a^2a + 2a^2a + b = 0$

$a^3 - 3a^3 + 2a^3 + b = 0$

$3a^3 - 3a^3 + b = 0$

b = 0

61. If x + 2 and x - 1 are the factors of $x^3 + 10x^2 + mx + n$, then the values of m and n are respectively

a. 5 and -3
b. 17 and -8
c. 7 and -18

d. 23 and -19

Ans. :

c. 7 and -18

Solution:

If (x + 2) and (x - 1) are factors of polynomial $x^3 + 10x^2 + mx + n$,

then x = -2, x = +1 will satisfy the polynomial.

Let $p(x) = x^3 + 10x^2 + mx + n$

Then, p(-2) = 0

$(-2)^3 + 10(-2)^2 + m(-2) + n = 0$

-8 + 40 - 2m + n = 0

32 - 2m + n = 0 ...(1)

And, p(1) = 0

$(1)^3 + 10(1)^2 + m(1) + n = 0$

1 + 10 + m + n = 0

11 + m + n = 0 ...(2)

Substracting equation (1) from equation (2), we get

-21 + 3m = 0

3m = 21

m = 7

Substituting m = 7 in equation (2),

11 + 7 + n = 0

18 + n = 0

n = -18

62. If $(x^{100} + 2x^{99} + k)$ is divisible by (x + 1) then the value of k is:

a. 1
b. 2
c. -2
d. -3

Ans. :

a. 1

Solution:

$p(x) = x^{100} + 2x^{99} + k$

$x + 1 = 0 \Rightarrow x = -1$

By the factor theorem, we know that when p(x) is divided by (x + 1), the remainder is p(-1).

Now, $p(-1) = (-1)^{100} + 2(-1)^{99} + k$

$\Rightarrow 0 = 1 - 2 + k$ ...(Given that p(x) is divisible by x + 1.)

$\Rightarrow k = 1$

63. If (x + 1) is a factor of the polynomial $(2x^2 + kx)$ then k = ?

a. 4
b. -3
c. 2

d. -2

Ans. :

c. 2

Solution:

Let p(x) = $2x^2 + kx$

Since (x + 1) is a factor of p(x),

= P(-1) = 0

⇒ $2(-1)^2 + k(-1) = 0$

⇒ 2 - k = 0

⇒ k = 2

64. If (x + 2) and (x - 1) are factors of the polynomial p(x) = x3 + 10x2 + mx + n then:

a. m = 5, n = -3
b. m = 7, n = -18
c. m = 17, n = -8
d. m = 23, n = -19

Ans. :

b. m = 7, n = -18

Solution:

Let f(x) = $x^3 + 10x^2 + mx + n$

Now, x + 2 = 0 ⇒ x = -2

and x - 1 = 0 ⇒ x = 1

By factor theorem,

f(-2) = 0

⇒ $(-2)^3 + 10(-2)^2 + m(-2) + n$

⇒ -8 + 40 - 2m + n = 0

⇒ 2m - n = 32 ...(i)

By factor theorem,

f(1) = 0

⇒ $(1)^3 + 10(1)^2 + m(1) + n = 0$

⇒ m + n = -11 ...(ii)

Adding (i) and (ii), we get

3m = 21

⇒ m = 7

Substituting in (ii), we get

n = -18

65. For what value of k is the polynomial p(x) = $2x^3 - kx^2 + 3x + 10$ exactly divisible by (x + 2)?

a. $-\frac{1}{3}$
b. $\frac{1}{3}$
c. 3
d. -3

Ans. :

d. -3

Solution:

p(x) = $2x^3 - kx^2 + 3x + 10$

x + 2 = 0 ⇒ x = -2

By the factor theorem, we know that when p(x) is divided by (x + 2), the remainder is p(-2).

Now, p(-2) = $2(-2)^3 + k(-2)^2 + 3(-2) + 10$

⇒ 0 = -16 - 4k - 6 + 10

⇒ 0 = -12 - 4k

⇒ 4k = -12

⇒ k = -3

66. If (x + 5) is a factor of = $x^3 - 20x + 5k$ then k = ?

a. -5
b. 5
c. 3
d. -3

Ans. :

b. 5

Solution:

p(x) = $x^3 - 20x + 5k$

Now, x + 5 = 0 ⇒ x = (-5)

By factor theorem,

p(-5) = 0

⇒ $(-5)^3 - 20(-5) + 5k = 0$

⇒ -125 + 100 + 5k = 0

⇒ -25 + 5k = 0

⇒ 5k = 25

⇒ k = 5

*** A statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.** **[5]**

67. **Directions:** In the following questions, the Assertions (A) and Reason(s) (R) have been put forward. Read both the statements carefully and choose the correct alternative from the following:

Assertion: The LCM of $(x^2 + x - 6)$ and $4\ (4 - x)^2$ is 4 (x + 3) (x + 2) (x - 2)

Reason: $x^{100} + 2x^{99} + k$ is divisible by (x + 1) then the value of k is 2.

a. Both Assertion and Reason are correct and Reason is the correct explanation for Assertion.
b. Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.
c. Assertion is true but the reason is false.
d. Both assertion and reason are false.

Ans. :

c. Assertion is true but the reason is false.

68. **Directions:** In the following questions, the Assertions (A) and Reason(s) (R) have been put forward. Read both the statements carefully and choose the correct alternative from the following:

Assertion: If (x + 2) is a factor of $x^3 - 2ax^2 + 16$ the value of a is 7.

Reason: If one of the factor of $x^2 + x - 20$ is (x + 5)and other is (x + 4).

a. Both Assertion and Reason are correct and Reason is the correct explanation for Assertion.
b. Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.
c. Assertion is true but the reason is false.
d. Both assertion and reason are false.

Ans. :

d. Both assertion and reason are false.

69. **Directions:** In the following questions, the Assertions (A) and Reason(s) (R) have been put forward. Read both the statements carefully and choose the correct alternative from the following:

Assertion: $y^2 - 5$ is a quadratic polynomial.

Reason: Degree of polynomial 2 is called quadratic polynomial.

a. Both Assertion and Reason are correct and Reason is the correct explanation for Assertion.
b. Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.
c. Assertion is true but the reason is false.
d. Both assertion and reason are false.

Ans. :

a. Both Assertion and Reason are correct and Reason is the correct explanation for Assertion.

70. **Directions:** In the following questions, the Assertions (A) and Reason(s) (R) have been put forward. Read both the statements carefully and choose the correct alternative from the following:

Assertion: If one zero of polynomial $p(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of the other, then k = 2.

Reason: Lrrational zeros always occurs in pairs.

a. Both Assertion and Reason are correct and Reason is the correct explanation for Assertion.
b. Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.
c. Assertion is true but the reason is false.
d. Both assertion and reason are false.

Ans. :

b. Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.

71. **Directions:** In the following questions, the Assertions (A) and Reason(s) (R) have been put forward. Read both the statements carefully and choose the correct alternative from the following:

Assertion: A quadratic polynomial can have at most two zero.

Reason: $x^2 + 7x + 9$ has two zero.

a. Both Assertion and Reason are correct and Reason is the correct explanation for Assertion.

b. Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.

c. Assertion is true but the reason is false.

d. Both assertion and reason are false.

Ans. :

c. Assertion is true but the reason is false.

*** Answer the following questions in one sentence. [1 Marks Each] [7]**

72. Factorise: $\frac{25}{4}x^2 - \frac{y^2}{9}$.

Ans. : $a^2-b^2 = (a+b)(a-b)$

$\frac{25x^2}{4} - \frac{y^2}{9} = \left(\frac{5}{2}x\right)^2 - \left(\frac{y}{3}\right)^2$

$= \left(\frac{5}{2}x + \frac{y}{3}\right)\left(\frac{5}{2}x - \frac{y}{3}\right)$

73. If $x + y + z = 0$ then show that $x^3 + y^3 + z^3 = 3xyz$.

Ans. : We know that

$x^3 + y^3 + z^3 - 3xyz = (x+ y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

(Using Identity $a^3+ b^3+ c^3- 3abc= (a + b + c)(a^2+ b^2+ c^2- ab - bc - ca)$)

$= (0)(x^2 + y^2 + z^2 - xy - yz - zx)$ ($\because x + y + z = 0$)

$= 0$

$\Rightarrow x^3 + y^3+ z^3 = 3xyz$.

74. Factorise :$64m^3 - 343n^3$

Ans. : $64m^3 - 343n^3$

$= (4m)^3 - (7n)^3 = (4m - 7n)\{(4m)^2 + (4m)(7n) + (7n)^2\}$

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

75. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by x + 1

Ans. : x + 1

We need to find the zero of the polynomial x + 1

$x + 1 = 0 \Rightarrow x = -1$

While applying the remainder theorem, we need to put the zero of the polynomial x + 1 in the polynomial

$x^3 + 3x^2 + 3x + 1$,to get

$p(x) = x^3 + 3x^2 + 3x + 1$

$p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$

$=-1+3-3+1$

$= 0$

Therefore, we conclude that on dividing the polynomial x^3+3x^2+3x+1 by x + 1, we will get the remainder as 0.

76. Write the coefficient of x^2 in $\frac{\pi}{2}x^2+x$

Ans. : $\frac{\pi}{2}x^2+x$
The coefficient of x^2 in the polynomial $\frac{\pi}{2}x^2+x$ is $\frac{\pi}{2}$.

77. Verify whether the following are True or False:
$\frac{-4}{5}$ is a zero of 4 - 5y

Ans. : False

Solution:

Because zero of 4 - 5y is $\frac{4}{5}$. $\left[\therefore 4 - 5y = 0 \Rightarrow y = \frac{4}{5}\right]$

78. Which of the following expression are polynomials?
$\frac{1}{7}a^3-\frac{2}{\sqrt{3}}a^2+4a-7$

Ans. : Polyonimial, because the exponent of the variable of $\frac{1}{7}a^3-\frac{2}{\sqrt{3}}a^2+4a-7$ is a whole number.

*** Answer the following short questions. [2 Marks Each] [50]**

79. Find the value of k, if x – 1 is a factor of $4x^3 + 3x^2 - 4x + k$.

Ans. : As x – 1 is a factor of $p(x) = 4x^3 + 3x^2 - 4x + k$, therefore,
p(1) = 0
Now, $p(1) = 4(1)^3 + 3(1)^2 - 4(1) + k$
So, 4 + 3 – 4 + k = 0
i.e., k = –3

80. Verify : $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Ans. : We know that
$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ {Using Identity $(a + b)^3= a^3+ b^3+ 3ab (a + b)$}
$\Rightarrow x^3 + y^3 = (x + y)^3 - 3xy(x + y)$
$\Rightarrow x^3 + y^3 = (x + y)\{(x + y)^2 - 3xy\}$
$\Rightarrow x^3 + y^3 = (x + y)(x^2 + 2xy + y^2 - 3xy)$ {Using Identity $(a + b)^2= a^2+ 2ab + b^2$ }
$\Rightarrow x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

81. Verify : $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Ans. : We know that
$(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ {Using Identity $(a - b)^3= a^3- b^3- 3ab (a - b)$}
$\Rightarrow x^3 - y^3 = (x - y)^3 + 3xy(x - y)$
$\Rightarrow x^3 - y^3 = (x - y)\{(x - y)^2 + 3xy\}$
$\Rightarrow x^3 - y^3 = (x - y)(x^2 - 2xy + y^2 + 3xy)$
$\Rightarrow x^3 - y^3 = (x - y)(x^2 + xy+ y^2)$

82. Factorise : $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$.

Ans. : $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

= $(3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)\left(\frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

= $\left(3p - \frac{1}{6}\right)^3$

(Using Identity $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$)

= $\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

83. Verify $x = -\frac{m}{l}$ are zeroes of the polynomial $p(x) = lx + m$

Ans. : $p(x) = lx + m,\ x = -\frac{m}{l}$

We need to check whether $p(x) = lx + m$ at $x = -\frac{m}{l}$ is equal to zero or not,

i.e., $p\left(\frac{-m}{l}\right)$ is equal to zero or not.

$p\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right) + m\ = -m + m\ = 0$

Therefore, $x = -\frac{m}{l}$ is a zero of the polynomial $p(x) = lx + m$

84. Expand the following:

$\left(4 - \frac{1}{3\mathrm{x}}\right)^3$

Ans. : $\left(4 - \frac{1}{3\mathrm{x}}\right)^3 = (4)^3 + \left(-\frac{1}{3\mathrm{x}}\right)^3 + 3(4)\left(-\frac{1}{3\mathrm{x}}\right)\left(4 - \frac{1}{3\mathrm{x}}\right)$

[Using identity, $(a - b)^3 = a^3 - b^3 + 3a(-b)(a - b)$]

$= 64 - \frac{1}{27\mathrm{x}^3} - \frac{4}{\mathrm{x}}\left(4 - \frac{1}{3\mathrm{x}}\right)$

$= 64 - \frac{1}{27\mathrm{x}^3} - \frac{16}{\mathrm{x}} + \frac{4}{3\mathrm{x}^2}$

85. Without actually calculating the cubes, find the value of:

$(0.2)^3 - (0.3)^3 + (0.1)^3$

Ans. : Given, $(0.2)^3 - (0.3)^3 + (0.1)^3$ or $(0.2)^3 + (-0.3)^3 + (0.1)^3$

Here, we see that, 0.2 - 0.3 + 0.1 = 0.3 - 0.3 = 0

$\therefore (0.2)^3 - (0.3)^3 + (0.1)^3 = 3 \times (0.2) \times (-0.3) \times (0.1)$

[Using identity, if a + b + c = 0, then $a^3 + b^3 + c^3 = 3abc$]

= -0.6 × 0.003 = -0.018

86. By Remainder Theorem find the remainder, when p(x) is divided by g(x), where:

$p(x) = x^3 - 2x^2 - 4x - 1$, $g(x) = x + 1$

Ans. : Given, $p(x) = x^3 - 2x^2 - 4x - 1$ and $g(x) = x + 1$

Here, zero of g(x) is -1.

When we divide p(x) by g(x) using remainder theorem, we get the remainder p(-1)

$\therefore p(-1) = (-1)^3 - 2(-1)^2 - 4(-1) - 1$

= -1 - 2 + 4 - 1

= 4 - 4 = 0

Hence, remainder is 0.

87. By Remainder Theorem find the remainder, when p(x) is divided by g(x), where:
$p(x) = 4x^3 - 12x^2 + 14x - 3$, $g(x) = 2x - 1$

Ans. : Given, $p(x) = 4x^3 - 12x^2 + 14x - 3$, $g(x) = 2x - 1$

Here, zero of g(x) is $\frac{1}{2}$.

When we divide p(x) by g(x) using remainder theorem, we get the remainder $\mathrm{p}\left(\frac{1}{2}\right)$

$$\therefore \mathrm{p}\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 14\left(\frac{1}{2}\right) - 3$$
$$= 4 \times \frac{1}{8} - 12 \times \frac{1}{4} + 14 \times \frac{1}{2} - 3$$

Hence, remainder is $\frac{3}{2}$.

88. For what value of m is $x^3 - 2mx^2 + 16$ divisible by x + 2 ?

Ans. : Let $p(x) = x^3 - 2mx^2 + 16$

Since, p(x) is divisible by (x + 2), then remainder = 0

P(-2) = 0

$\Rightarrow (-2)^3 - 2m(-2)^2 + 16 = 0$

$\Rightarrow -8 - 8m + 16 = 0$

$\Rightarrow 8 = 8m$

m = 1

Hence, the value of m is 1 .

89. Factorise:
$$2\sqrt{2}\mathrm{a}^3 + 8\mathrm{b}^3 - 27\mathrm{c}^3 + 18\sqrt{2}\mathrm{abc}.$$

Ans. : We have,

$$2\sqrt{2}\mathrm{a}^3 + 8\mathrm{b}^3 - 27\mathrm{c}^3 + 18\sqrt{2}\mathrm{abc}.$$
$$= \left\{(\sqrt{2}\mathrm{a})^3 + (2\mathrm{b})^3 + (-3\mathrm{c})^3 - 3(\sqrt{2}\mathrm{a})(2\mathrm{b})(-3\mathrm{c})\right\}$$
$$= \left\{\sqrt{2}\mathrm{a} + 2\mathrm{b} - 3\mathrm{c}\right\}\left\{(\sqrt{2}\mathrm{a})^2 + (2\mathrm{b})^2 + (-3\mathrm{c})^2 - (\sqrt{2}\mathrm{a})(2\mathrm{b}) - (2\mathrm{b})(-3\mathrm{c})(\sqrt{2}\mathrm{a})\right\}$$
$$= (\sqrt{2}\mathrm{a} + 2\mathrm{b} - 3\mathrm{c})(2\mathrm{a}^2 + 4\mathrm{b}^2 + 9\mathrm{c}^2 - 2\sqrt{2}\mathrm{ab} + 6\mathrm{bc} + 3\sqrt{2}\mathrm{ca})$$

90. If a, b, c are all non-zero and a + b + c = 0, prove that $\frac{\mathrm{a}^2}{\mathrm{bc}} + \frac{\mathrm{b}^2}{\mathrm{ca}} + \frac{\mathrm{c}^2}{\mathrm{ab}} = 3.$

Ans. : We have a, b, c are all non-zero and a + b + c = 0, therefore

$a^3 + b^3 + c^3 = 3abc$

Now, $\frac{\mathrm{a}^2}{\mathrm{bc}} + \frac{\mathrm{b}^2}{\mathrm{ca}} + \frac{\mathrm{c}^2}{\mathrm{ab}} = \frac{\mathrm{a}^3+\mathrm{b}^3+\mathrm{c}^3}{\mathrm{abc}} = \frac{3\mathrm{abc}}{\mathrm{abc}} = 3$

91. Find the following:
$(x^2 - 1)(x^4 + x^2 + 1)$

Ans. : We have,

$(x^2 - 1)(x^4 + x^2 + 1)$

$= (x^2 - 1)\left[(x^2)2 + 1 \times x^2 + 1^2\right]$

$= (x^2)^3 - (1)^3 \left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)\right]$

$= x^6 - 1$

$\therefore (x^2 - 1)(x^4 + x^2 + 1) = x^6 - 1$

92. Write the following in the expanded form:

$$\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)^2$$

Ans. : We have,

$$\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)^2$$

$$=\left(\frac{x}{y}\right)^2+\left(\frac{y}{z}\right)+\left(\frac{z}{x}\right)^2$$

$$+2\frac{x}{y}\frac{y}{z}+2\frac{y}{z}\frac{z}{x}+2\frac{z}{x}\frac{x}{y}$$

$$[\therefore (x+y+z)$$

$$=x^2+y^2+z^2+2xy+2yz+2xz]$$

$$\therefore \left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)^2$$

$$=\left(\frac{x^2}{y^2}\right)+\left(\frac{y^2}{z^2}\right)+\left(\frac{z^2}{x^2}\right)+2\frac{x}{z}+2\frac{y}{x}+2\frac{x}{y}$$

93. Evaluate the following using identities:

$$\left(1.5x^2-0.3y^2\right)\left(1.5x+0.3y^2\right)$$

Ans. : We have,

$$\left(1.5x^2-0.3y^2\right)\left(1.5x+0.3y^2\right)$$

$$=\left[1.5x^2\right]^2-\left[0.3y^2\right]^2\left[\begin{array}{c}\because (a+b)(a-b)=a^2-b^2\\ a=1.5x^2 \text{ and } b=0.3y^2\end{array}\right]$$

$$=2.25x^4-0.09y^4$$

$$=\left[1.5x^2-0.3y^2\right]\left[1.5x^2+0.3y^2\right]$$

$$=2.25x^4-0.09y^4.$$

94. Simplify the following:

$$\frac{7.83\times 7.83-1.17\times 1.17}{6.66}$$

Ans. : We have,

$$\frac{7.83\times 7.83-1.17\times 1.17}{6.66}$$

$$=\frac{(7.83+1.17)(7.83-1.17)}{6.66}\left[\because (a-b)^2=(a+b)(a-b)\right]$$

$$=\frac{(9.00)(6.66)}{6.66}=9$$

$$\therefore \frac{7.83\times 7.83-1.17\times 1.17}{6.66}=9$$

95. Write the following in the expanded form:

$$\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2$$

Ans. : We have,

$$\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2=\left(\frac{a}{bc}\right)^2+\left(\frac{b}{ca}\right)^2+\left(\frac{c}{ab}\right)^2$$

$$+2\left(\frac{a}{bc}\right)\left(\frac{b}{ca}\right)+2\left(\frac{b}{ca}\right)\left(\frac{c}{ab}\right)+2\left(\frac{a}{bc}\right)\left(\frac{c}{ab}\right)$$

$\left[\therefore \left(x+y+z\right) = x^2+y^2+z^2+2xy+2yz+2xz\right]$

$\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2$

$=\left(\frac{a^2}{b^2c^2}\right)+\left(\frac{b^2}{c^2a^2}\right)+\left(\frac{c^2}{a^2b^2}\right)+2\frac{2}{a^2}+\frac{2}{b^2}+\frac{2}{c^2}$

96. Factorize the following expressions:

$(2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3$

Ans. : $(2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3$

Let $2x - 3y = a$, $4z - 2x = b$, $3y - 4z = c$

$\therefore a + b + c = 2x - 3y + 4z - 2x + 3y - 4z = 0$

$\because a + b + c = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$\therefore (2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3$

$= 3(2x - 3y)(4z - 2x)(3y - 4z)$

97. Factorize the following expressions:

$2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc$

Ans. : $2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc$

$=\left(\sqrt{2}a\right)^3+\left(\sqrt{3}b\right)^3+c^3-3\times\sqrt{2}a\times\sqrt{3}b\times c$

$=\left(\sqrt{2}a+\sqrt{3}b+c\right)\Big(\left(\sqrt{2}a\right)^2+\left(\sqrt{3}b\right)^2+$

$c^2-\left(\sqrt{2}a\right)\left(\sqrt{3}b\right)-\left(\sqrt{3}b\right)c-\left(\sqrt{2}a\right)c\Big)$

$=\left(\sqrt{2}a+\sqrt{3}b+c\right)\left(2a^2+3b^2+c^2-\sqrt{6}ab-\sqrt{3}bc-\sqrt{2}ac\right)$

$\therefore 2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc$

$=\left(\sqrt{2}a+\sqrt{3}b+c\right)\left(2a^2+3b^2+c^2-\sqrt{6}ab-\sqrt{3}bc-\sqrt{2}ac\right)$

98. Factorize:

$\frac{8}{27}x^3+1+\frac{4}{3}x^2+2x$

Ans. : $\frac{8}{27}x^3+1+\frac{4}{3}x^2+2x$

$=\left(\frac{2}{3}x\right)^3+(1)^3+3\times\left(\frac{2}{3}x\right)^2\times 1+3(1)^2\times\left(\frac{2}{3}x\right)$

$=\left(\frac{2}{3}x+1\right)^3$ $\left[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3\right]$

$=\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)$

$\therefore \frac{8}{27}x^3+1+\frac{4}{3}x^2+2x$

$=\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)$

99. Multiply:

$(x^2 + y^2 + z^2 - xy + xz + yz)$ by $(x + y -z)$

Ans. : $(x^2 + y^2 + z^2 - xy + xz + yz)$ by $(x + y -z)$

$= (x + y - z)(x^2 + y^2 + z^2 - xy + xz + yz)$

$= (x + y + (-z))(x^2 + y^2 + (-z)^2 - xy - y(-z) - (-z)x)$

$= x^3 + y^3 + (-z)^3 - 3xyz(-z)$ $\left[\because (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc\right]$

$= x^3 + y^3 - z^3 + 3xyz$

100. If $a^2 + b^2 + c^2 = 250$ and $ab + bc + ca = 3$, find $a + b + c$.

Ans. : Recall the formula

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Given that

$a^2 + b^2 + c^2 = 250$, $ab + bc + ca = 3$

Then we have

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$(a + b + c)^2 = 250 + 2.(3)$

$(a + b + c)^2 = 256$

$(a + b + c) = \pm 16$

101. In the following, use factor theorem to find whether polynomial g(x) is a factor of polynomial f(x) or, not:

$f(x) = 3x^4 + 17x^3 + 9x^2 - 7x - 10$; $g(x) = x + 5$

Ans. : Let $g(x) = 0$

$\Rightarrow x + 5 = 0$

$\Rightarrow x = -5$

Now,

$f(-5) = 3(-5)^4 + 17(-5)^3 + 9(-5)^2 - 7(-5) - 10$

$= 3(625) + 17(-125) + 9(25) + 35 - 10$

$= 1875 - 2125 + 225 + 35 - 10$

$\because f(-5) = 0$, by factor theorem $x + 5$ is a factor of f(x).

102. What must be subtracted from $x^3 - 6x^2 - 15x + 80$ so that the result is exactly divisible by $x^2 + x - 12$?

$$
\begin{array}{r|l}
 & \mathrm{x - 7} \\
\mathrm{x^2 + x - 12} & \mathrm{x^3 - 6x^2 - 15x + 80} \\
 & \mathrm{x^3 + x^2 - 12x} \\
 & \mathrm{- \quad - \quad +} \\
\hline
 & \mathrm{-7x^2 - 3x + 80} \\
 & \mathrm{-7x^2 - 7x + 84} \\
\hline
 & \mathrm{Remainder = 4x - 4}
\end{array}
$$

Ans. :

$\therefore 4x - 4$ should subtracted from $x^3 - 6x^2 - 15x + 80$

So that the result is exactly divisible by $x^2 + x - 12$.

103. Factorise:

$\sqrt{2}x^2 + 3x + \sqrt{2}$

Ans. : $\sqrt{2}x^2 + 3x + \sqrt{2}$

$= \sqrt{2}x^2 + x + 2x + \sqrt{2}$

$= x(\sqrt{2}x + 1) + \sqrt{2}(\sqrt{2}x + 1)$

$= (\sqrt{2}x + 1)(x + \sqrt{2})$

* Answer the following questions. [3 Marks Each] [57]

104. If a + b + c = 5 and ab + bc + ca = 10, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

Ans. : To prove, $a^3 + b^3 + c^3 - 3abc = -25$

Given,

a + b + c = 5, ab + bc + ca = 10

$\therefore (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\therefore (5)^2 = a^2 + b^2 + c^2 = 25(10)$

$\Rightarrow 25 = a^2 + b^2 + c^2 = 20$

$\Rightarrow a^2 + b^2 + c^2 = 25 - 20$

$\Rightarrow a^2 + b^2 + c^2 = 5$

LHS = $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

= (5)[5 - (ab + bc + ca)]

= 5(5 - 10) = 5(-5) = -25 = RHS

Hence proved.

105. If a + b + c = 9 and ab + bc + ca = 26, find $a^2 + b^2 + c^2$.

Ans. : We have that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + bc + 2ca$

$\Rightarrow (a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca)$

$\Rightarrow 9^2 = (a^2 + b^2 + c^2) + 2(26)$

[Putting the value of a + b + c and ab + bc + ca]

$\Rightarrow 81 = (a^2 + b^2 + c^2) + 52$

$\Rightarrow (a^2 + b^2 + c^2) = 81 - 52 = 29$

106. For the polynomial $\frac{x^3+2x+1}{5} - \frac{7}{2}x^2 - x^6$, write.

i. The degree of the polynomial.
ii. The coefficient of x^3.
iii. The coefficient of x^6.
iv. The constant term.

Ans. : $\frac{x^3+2x+1}{5} - \frac{7}{2}x^2 - x^6$

$\frac{x^3}{5} + \frac{2x}{5} + \frac{1}{5} - \frac{7}{2}x^2 - x^6$

i. We know that highest power of variable in a polynomial is the degree of the polynomial. In the given polynomial, the term with highest of x is - x, and the exponent of x in this term in 6.
ii. The coefficient of x^3 is $\frac{1}{5}$
iii. The coefficient of x^6 is -1.
iv. The constant term is $\frac{1}{5}$

107. If x = 3 and y = -1, find the values of the following using in identity:

$$\left(\frac{x}{4}-\frac{y}{3}\right)\left(\frac{x^2}{16}+\frac{xy}{12}+\frac{y^2}{9}\right)$$

Ans. : We have,

$$\left(\frac{x}{4}-\frac{y}{3}\right)\left(\frac{x^2}{16}+\frac{xy}{12}+\frac{y^2}{9}\right)$$

$$=\left(\frac{x}{4}-\frac{y}{3}\right)\left[\left(\frac{x}{4}\right)^2+\frac{x}{4}\times\frac{y}{3}+\left(\frac{y}{3}\right)^2\right]$$

$$=\left(\frac{x}{4}\right)^3-\left(\frac{y}{3}\right)^3\ \left[\because a^3-b^3=(a-b)(a^2+ab+b^2)\right]$$

$$=\frac{x^3}{64}-\frac{y^3}{27}$$

$$=\frac{(3)^3}{64}-\frac{(-1)^3}{27}\ \left[\because x=3,\ y=-1\right]$$

$$=\frac{27}{64}+\frac{1}{27}$$

$$=\frac{729+64}{1728}=\frac{793}{1728}$$

$$\therefore\left(\frac{x}{4}-\frac{y}{3}\right)\left(\frac{x^2}{16}+\frac{xy}{12}+\frac{y^2}{9}\right)=\frac{793}{1728}$$

108. Find the following products:

$(2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca)$

Ans. : We have,

$(2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca)$

$= (2a + (-3b) + (-2c)) + \left((2a)^2 + (-3b)^2 + (-2c)^2 - (2a)(-3b)(-2c) - (-2c)(2a)\right)$

$= (2a)^3 + (-3b)^3 + (-2c)^3 - 3(2a)(-3b)(-2c)$

$\left[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)\right]$

$= 8a^3 - 27b^3 - 8c^3 - 36abc$

$\therefore (2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca) = 8a^3 - 27b^3 - 8c^3 - 36abc$

109. If $a^2 + b^2 + c^2 = 16$ and $ab + bc + ca = 10$, find the value of $a + b + c$.

Ans. : We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$(x + y + z)^2 = 16 + 2(10)$

$(x + y + z)^2 = 36$

$$(x+y+z)=\sqrt{36}$$

$$(x+y+z)=\pm 6$$

Hence, value of required expression $(a+b+c)=\pm 6$

110. If x = 3 and y = -1, find the values of the following using in identity:

$$\left(\frac{x}{7}+\frac{y}{3}\right)\left(\frac{x^2}{49}+\frac{y^2}{9}-\frac{xy}{21}\right)$$

Ans. : We have,

$$\left(\frac{x}{7}+\frac{y}{3}\right)\left(\frac{x^2}{49}+\frac{y^2}{9}-\frac{xy}{21}\right)$$

$$= \left(\frac{x}{7} + \frac{y}{3}\right)\left[\left(\frac{x}{7}\right)^2 + \left(\frac{y}{3}\right) - \frac{x}{7} \times \frac{y}{3}\right]$$
$$= \left(\frac{x}{7}\right)^3 + \left(\frac{y}{3}\right)^3 \left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)\right]$$
$$= \frac{x^3}{343} + \frac{y^3}{27}$$
$$= \frac{(3)^3}{343} + \frac{(-1)^3}{27} \left[\because x = 3 \text{ and } y = -1\right]$$
$$= \frac{27}{343} + \frac{-1}{27}$$
$$= \frac{729-343}{9261} = \frac{386}{9261}$$
$$\therefore \left(\frac{x}{7} + \frac{y}{3}\right)\left(\frac{x^2}{49} + \frac{y^2}{9} - \frac{xy}{21}\right) = \frac{386}{9261}$$

111. If x = -2 and y = 1, by using an identity find the value of the following:

$$\left(5y + \frac{15}{y}\right)\left(25y^2 - 75 + \frac{225}{y^2}\right)$$

Ans. : We have,

$$\left(5y + \frac{15}{y}\right)\left(25y^2 - 75 + \frac{225}{y^2}\right)$$
$$\left(5y + \frac{15}{y}\right)\left[\left(5y\right)^2 - 5y \times \frac{15}{y}\left(\frac{15}{y}\right)^2\right]$$
$$= (5y)^3 + \left(\frac{15}{y}\right)^3 \left[\therefore a^3 + b^3 = (a+b)(a^2 - ab + b^2)\right]$$
$$= 125y^3 + \frac{3375}{y^3}$$
$$= 125(1)^3 + \frac{3375}{(1)^3} \left[\therefore y = 1\right]$$
$$= 125 + 3375$$
$$= 3500$$
$$\therefore \left(5y + \frac{15}{y}\right)\left(25y^2 - 75 + \frac{225}{y^2}\right) = 3500$$

112. Simplify:

$$\frac{173\times173\times173+127\times127\times127}{173\times173-173\times127+127\times127}$$

Ans. : $\frac{173\times173\times173+127\times127\times127}{173\times173-173\times127+127\times127}$

$$= \frac{173^3+127^3}{173^2-173\times127+127^2}$$
$$= \frac{(173+127)(173^2-173\times127+127^2)}{173^2-173\times127+127^2}$$

$\left[\therefore a^3 + b^3 = (a + b)(a^2 - ab + b^2)\right]$

= (173 + 127)

= 300

113. Factorize the following expressions:

$a^3 + 3a^2b + 3ab^2 + b^3 - 8$

Ans. : $= (a + b)^3 - 8$

$\left[\therefore a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)3\right]$

$= (a + b)^3 - 23$

$= (a + b - 2)((a + b)^2 + (a + b) \times 2 + 2^2)$

$= (a + b - 2)(a^2 + 2ab + b^2 + 2a + 2b + 4)$

$\therefore a^3 + 3a^2b + 3ab^2 + b^3 - 8 = (a + b - 2)(a^2 + 2ab + b^2 + 2a + 2b + 4)$

114. Factorize:

$xy^9 - yx^9$

Ans. : The given expression to be factorized is

$xy^9 - yx^9$

This can be wriiten in the form

$xy^9 - yx^9 = x.y.y^8 - y.x.x^8$

Take common xy from the two terms of the above expression

$xy^9 - yx^9 = xy(y^8 - x^8)$

$= xy(y^8 - x^8)$

$= \{xy(y^4)^2 - (x^4)^2)\}$

$= xy(y^4 + x^4)(y^4 - x^4)$

$xy^9 - yx^9 = xy(y^4 + x^4)\{(y^2)^2 - (x^2)^2\}$

$= xy(y^4 + x^4)(y^2 + x^2)(y^2 - x^2)$

$= xy(y^4 + x^4)(y^2 + x^2)\{(y)^2 - (x)^2\}$

$= xy(y^4 + x^4)(y^2 + x^2)(y + x)(y - x)$

We cannot further factorize the expression.

So, the required factorization of $xy^9 - yx^9$ is $xy(y^4 + x^4)(y^2 + x^2)(y + x)(y - x)$

115. Factorize:

$(a - b + c)^2 + (b - c + a)^2 + 2(a - b + c)(b - c + a)$

Ans. : $(a - b + c)^2 + (b - c + a)^2 + 2(a - b + c)(b - c + a)$

Let $(a - b + c) = x$ and $(b - c + a) = y$

$= x^2 + y^2 + 2xy$

Using the identity $(a + b)^2 = a^2 + b^2 + 2ab$

$= (x + y)^2$

Now, substituting x and y

$(a - b + c + b - c + a)^2$

Cancelling -b, +b & + c, -c

$= (2a)^2$

$= 4a^2$

$\therefore (a - b + c)^2 + (b - c + a)^2 + 2(a - b + c)(b - c + a) = 4a^2$

116. Multiply:

$(9x^2 + 25y^2 + 15xy + 12x - 20y + 16)$ by $(3x - 5y + 4)$

Ans. : $= (3x - 5y + 4)(9x^2 + 25y^2 + 15xy + 20y - 12x + 16)$

$= (3x + (5y) + 4)\{(3x)^2 + (-5y)^2 + 4^2 - 3x(-5y) - (-5y)4 - 4(3x)\}$

$\left[\because (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc\right]$

Here, a = 3x, b = -5y, c = 4

$= (3x)^3 + (-5y)^3 + 4^3 - 3(3x)(-5y)(4)$

$= 27x^3 - 125y^3 + 64 + 180xy$

$\therefore (3x - 5y + 4)(9x^2 + 25y^2 + 15xy + 20y - 12x + 16)$

$= 27x^3 - 125y^3 + 64 + 180xy$

117. If x = 2 is a root of the polynomial $f(x) = 2x^2 - 3x + 7a$, Find the value of a.

Ans. : We know that, $f(x) = 2x^2 - 3x + 7a$

Given that x = 2 is the root of f(x)

Substitute the value of x in f(x)

$f(2) = 2(2)^2 - 3(2) + 7a$

$= (2 \times 4) - 6 + 7a$

$= 8 - 6 + 7a$

$= 7a + 2$

Now, equal 7a + 2 to zero

$\Longrightarrow 7a + 2 = 0$

$\Longrightarrow 7a = -2$

$\Longrightarrow a = -27$

The value of $a = -\frac{2}{7}$

118. What must be added to $3x^3 + x^2 - 22x + 9$ so that the result is exactly divisible by $3x^2 + 7x - 6$?

Ans. : We know that, Dividend = Divisor × Quotient + Remainder

Dividend = $3x^3 + x^2 - 22x + 9$

Divisor = $3x^2 + 7x - 6$

$$\begin{array}{r|l} & x - 2 \\ \hline 3x^2 + 7x - 6 & 3x^3 + x^2 - 22x + 9 \\ & 3x^3 + 7x^2 - 6x \\ & - \quad - \quad + \\ \hline & -6x^2 - 16x + 9 \\ & -6x^2 - 14x + 12 \\ & + \quad + \quad - \\ \hline & -2x - 3 \\ \hline \text{Remainder} = & -2x - 3 \end{array}$$

Remainder = -2x - 3

So, -(-2x - 3) = 2x + 3 should be added to $3x^3 + x^2 - 22x + 9$ to make it exactly divisible by $3x^2 + 7x - 6$.

119. Find the value k if x - 3 is a factor of $k^2x^3 - kx^2 + 3kx - k$.

Ans. : Let g(x) = 0

⇒ x - 3 = 0

⇒ x = 3,

Since (x - 3) is a factor of f(x)

$\therefore f(3) = 0$

$f(3) = k^2 3^3 - k3^2 + 3k(3) - k = 0$

$\Rightarrow 27k^2 - 9k + 9k - k = 0$

$\Rightarrow 27k^2 - k = 0$

$\Rightarrow k(27k - 1) = 0$

$\therefore k = 0, 27k - 1 = 0$

$27k = 1$

$k = \frac{1}{27}$

Hence k = 0, $k = \frac{1}{27}$

120. If x - 2 is a factor of the following two polynomials, find the values of a in case:
$x^3 - 2ax^2 + ax - 1$

Ans. : $x^3 - 2ax^2 + ax - 1$

Let,

$x - 2 = 0$

$\therefore x = 2$

$\because x - 2$ is a factor of $p(x) = x^3 - 2ax^2 + ax - 1$

$\therefore p(2) = 0$

$p(2) = 2^3 - 2a(2)^2 + 2a - 1 = 0$

$\Rightarrow 8 - 8a + 2a - 1 = 0$

$\Rightarrow 7 - 6a = 0$

$\Rightarrow 6a = 7$

$\Rightarrow \ a = \frac{7}{6}$

121. Find the value of *a* such that (x - 4) is a factors of $5x^3 - 7x^2 - ax - 28$.

Ans. : Let g(x) = x - 4, $f(x) = 5x^3 - 7x^2 - ax - 28$

Let g(x) = 0

$\Rightarrow x - 4 = 0$

$\Rightarrow x = 4,$

Since (x - 4) is a factor of f(x).

$\therefore f(4) = 0$

$f(4) = 5(4)^3 - 7(4)^2 - a(4) - 28 = 0$

$\Rightarrow 5(64) - 7(16) - 4a - 28 = 0$

$\Rightarrow 320 - 112 - 4a - 28 = 0$

$\Rightarrow 180 - 4a = 0$

$\Rightarrow 4a = 180$

$\Rightarrow \ a = \frac{180}{4} = 45$

122. Factorise:
$(5a - 7b)^3 + (7b - 9c)^3 + (9c - 5a)^3$

Ans. : Put (5a - 7b) = x, (7b - 9c) = y, (9c - 5a) = z.

Here,

$x + y + z = 5a - 7b + 9c - 5a + 7b - 9c = 0$

Thus,

We have:

$(5a - 7b)^3 + (9c - 5a)^3 + (7b - 9c)^3 = x^3 + z^3 + y^3$

$= 3xyz$ [When $x + y + z = 0$, $x^3 + y^3 + z^3 = 3xyz$]

$= 3(5a - 7b)(9c - 5a)(7b - 9c)$

* Questions with calculation. [4 Marks Each] [52]

123. The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by $x + 1$ leaves the remainder 19. Find the values of a. Also find the remainder when p(x) is divided by x + 2.

Ans. : We know that if p(x) is divided by x + a, then the remainder = p(-a).

Now, $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ is divided by $x + 1$, then the remainder = p(-1)

Now, $p(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + 3a - 7$

$= 1 - 2(-1) + 3(1) + a + 3a - 7$

$= 1 + 2 + 3 + 4a - 7$

$= -1 + 4a$

Also, remainder = 19

$\therefore -1 + 4a = 19$

$\Rightarrow 4a = 20; a = 20 \div 4 = 5$

Again, when p(x) is divided by x + 2, then

Remainder $= p(-2) = (-2)^4 - 2(-2)^3 + 3(-2)^2 - a(-2) + 3a - 7$

$= 16 + 16 + 12 + 2a + 3a - 7$

$= 37 + 5a$

$= 37 + 5(5) = 37 + 25 = 62$

124. Prove that $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$.

Ans. : $(a + b + c)^3 = [a + (b + c)]^3$

$= a^3 + 3a^2(b + c) + 3a(b + c)^2 + (b + c)^3$

$= a^3 + 3a^2b + 3a^2c + 3a(b^2 + 2bc + c^2) + (b^3 + 3b^2c + 3bc^2 + c^3)$

$= a^3 + 3a^2b + 3a^2c + 3ab^2 + 6abc + 3ac^2 + b^3 + 3b^2c + 3bc^2 + c^3$

$= a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3b^2c + 3c^2a + 3c^2b + 6abc$

$= a^3 + b^3 + c^3 + 3a^2(b + c) + a^3 + b^3 + c^3 + 3a^2(b + c)$

Hence, above result can be put in the form

$(a + b + c)^3 = (a + b + c)^3 + 3(a + b)(b + c)(c + a)$

$\therefore (a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$

125. Find the value of $27x^3 + 8y^3$, if:

$3x + 2y = 20$ and $xy = \frac{14}{9}$

Ans. : Given $3x + 2y = 20$, $xy = \frac{14}{9}$

On cubing both sides we get,

$(3x + 2y)^3 = (20)^3$

We shall use identity $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$27x^3 + 8y^3 + 3(3x)(2y)(3x + 2y) = 20 \times 20 \times 20$

$27x^3 + 8y^3 + 18(xy)(3x + 2y) = 8000$

$$27x^3 + 8y^3 + 18\left(\frac{14}{9}\right)(20) = 8000$$

$27x^3 + 8y^3 + 560 = 8000$

$27x^3 + 8y^3 = 8000 - 560$

$27x^3 + 8y^3 = 7440$

Hence the value of $27x^3 + 8y^3$ is 7440.

126. If $x^4 + \frac{1}{x^4} = 194$, find $x^3 + \frac{1}{x^3},\ x^2 + \frac{1}{x^2}$ and $x + \frac{1}{x}$.

Ans. : In the given problem, we have to find the value of $x^3 + \frac{1}{x^3},\ x^2 + \frac{1}{x^2},\ x + \frac{1}{x}$

Given $x^4 + \frac{1}{x^4} = 194$

By adding and subtracting $2 \times x^2 \times \frac{1}{x^2}$ in left hand side of $x^4 + \frac{1}{x^4} = 194$ we get,

$$x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2} - 2 \times x^2 \times \frac{1}{x^2} = 194$$

$$x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2} - 2 \times \left(\cancel{x}^2 \times \frac{1}{\cancel{x}^2}\right) = 194$$

$$\left(x^2 \times \frac{1}{x^2}\right)^2 - 2 = 194$$

$$\left(x^2 \times \frac{1}{x^2}\right)^2 = 194 + 2$$

$$\left(x^2 \times \frac{1}{x^2}\right)^2 = 196$$

$$\left(x^2 \times \frac{1}{x^2}\right)^2 = (14)^2$$

$$\left(x^2 \times \frac{1}{x^2}\right) = 14$$

Again by adding and subtracting $2 \times x \times \frac{1}{x}$ in left hand side of $\left(x^2 + \frac{1}{x^2}\right) = 14$ we get,

$$x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} - 2 \times x \times \frac{1}{x} = 14$$

$$\left(x + \frac{1}{x}\right)^2 - 2 \times \cancel{x} \times \frac{1}{\cancel{x}} = 14$$

$$\left(x + \frac{1}{x}\right)^2 - 2 = 14$$

$$\left(x + \frac{1}{x}\right)^2 = 14 + 2$$

$$\left(x + \frac{1}{x}\right)^2 = 16$$

$$\left(x + \frac{1}{x}\right)^2 = 4 \times 4$$

$$\left(x + \frac{1}{x}\right) = 4$$

Now cubing on both sides of $\left(x + \frac{1}{x}\right) = 4$ we get

$\left(x+\frac{1}{x}\right)^3=4^3$

we shall use identity $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x\times\frac{1}{x}\right)=4\times4\times4$

$x^3+\frac{1}{x^3}+3\times\cancel{x}\times\frac{1}{\cancel{x}}\times4=64$

$x^3+\frac{1}{x^3}+12=64$

$x^3+\frac{1}{x^3}=64-12$

$x^3+\frac{1}{x^3}=52$

Hence the value of $x^3+\frac{1}{x^3},\ x^2+\frac{1}{x^2},\ x+\frac{1}{x}$ is 52, 14, 4 respectively.

127. Factorize the following expressions:

$\left[\frac{x}{2}+y+\frac{z}{3}\right]^3+\left[\frac{x}{3}-\frac{2y}{3}+z\right]^3+\left[-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right]^3$

Ans. : $\left[\frac{x}{2}+y+\frac{z}{3}\right]^3+\left[\frac{x}{3}-\frac{2y}{3}+z\right]^3+\left[-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right]^3$

Let $\left(\frac{x}{2}+y+\frac{z}{3}\right)=a,\left(\frac{x}{3}-\frac{2y}{3}+z\right)=b,\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)=c$

$a+b+c=\frac{x}{2}+y+\frac{z}{3}+\frac{x}{3}-\frac{2y}{3}+z-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}$

$a+b+c=\left(\frac{x}{2}+\frac{x}{3}-\frac{5x}{6}\right)+\left(y-\frac{2y}{3}-\frac{y}{3}\right)+\left(\frac{z}{3}+z-\frac{4z}{3}\right)$

$a+b+c=\frac{3x}{6}+\frac{2x}{6}-\frac{5x}{6}+\frac{3y}{3}-\frac{2y}{3}-\frac{y}{3}+\frac{z}{3}+\frac{3z}{3}-\frac{4z}{3}$

$a+b+c=\frac{5x-5x}{6}+\frac{3y-3y}{3}+\frac{4z-4z}{3}$

$a+b+c=0$

$\because\ a+b+c=0$

$\therefore\ a^3+b^3+c^3=3abc$

$\therefore\left[\frac{x}{2}+y+\frac{z}{3}\right]^3+\left[\frac{x}{3}-\frac{2y}{3}+z\right]^3+\left[-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right]^3$

$=3\left(\frac{x}{2}+y+\frac{z}{3}\right)\left(\frac{x}{3}-\frac{2y}{3}+z\right)\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)$

128. Factorize the following expressions:

$(a + b)^3 - 8(a - b)^3$

Ans. : $= (a + b)^3 - [2(a - b)]^3$

$= (a + b)^3 - [2a - 2b]^3$

$= (a + b - (2a - 2b))((a + b)^2 + (a + b)(2a - 2b) + (2a - 2b)^2)$

$\therefore [a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$= (a + b - 2a + 2b)(a^2 + b^2 + 2ab + (a + b)(2a - 2b) + (2a - 2b)^2)$

$= (a + b - 2a + 2b)(a^2 + b^2 + 2ab + 2a^2 - 2ab + 2ab - 2b^2 + (2a - 2b)^2)$

$= (3b - a)(3a^2 + 2ab - b^2 + (2a - 2b)^2)$

$= (3b - a)(3a^2 + 2ab - b^2 + 4a^2 + 4b^2 - 8ab)$

$= (3b - a)(3a^2 + 4a^2 - b^2 + 4b^2 - 8ab + 2ab)$

$= (3b - a)(7a^2 + 3b^2 - 6ab)$

$\therefore (a + b)^3 - 8(a - b)^3 = (3b - a)(7a^2 + 3b^2 - 6ab)$

129. If the polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$, when divided by $(x - 2)$ leave the same remainder, Find the value of a.

Ans. : Here,

The polynomials are:

$f(x) = ax^3 + 3x^2 - 13$

$p(x) = 2x^3 - 5x + a$

equate, $x - 2 = 0$

$x = 2$

substitute the value of x in f(x) and p(x)

$f(2) = (2)^3 + 3(2)^2 - 13$

$= 8a + 12 - 13$

$= 8a - 1$ ...(1)

$p(2) = 2(2)^3 - 5(2) + a$

$= 16 - 10 + a$

$= 6 + a$ ...(2)

$f(2) = p(2)$

$\Rightarrow 8a - 1 = 6 + a$

$\Rightarrow 8a - a = 6 + 1$

$\Rightarrow 7a = 7$

$\Rightarrow a = 1$

The value of a = 1.

130. If both x + 1 and x - 1 are factors of $ax^3 + x^2 - 2x + b$, find the values of a and b.

Ans. : Let,

$x + 1 = 0$

$x = -1$

$\because (x + 1)$ is a factor of $p(x) = ax^3 + x^2 - 2x + b$

$\therefore p(-1) = 0$

$p(-1) = a(-1)^3 + (-1)^2 - 2(-1) + b = 0$

$\Rightarrow -a + 1 + 2 + b = 0$

$\Rightarrow -a + 3 + b = 0$

$\Rightarrow a = 3 + b$ ...(1)

Let,

$x - 1 = 0$

$x = 1$

$\because (x - 1)$ is a factor of p(x)

$\therefore p(1) = 0$

$p(1) = a(1)^3 + 1^2 - 2(1) + b = 0$

$\Rightarrow a + 1 - 2 + b = 0$

$\Rightarrow$ a = -b + 1 ...(2)

Equating (1) and (2)

$\Rightarrow$ 3 + b = -b + 1

$\Rightarrow$ b + b = 1 - 3

$\Rightarrow$ 2b = -2

$\Rightarrow$ b = -1

Substituting b = -1 in equation (2)

a = -(-1) + 1 = 1 + 1 = 2

$\therefore$ a = 2, b = -1

131. If the polynomial $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 4x + a$ leave the same remainder when divided by x - 2, Find the value of a.

Ans. : Given, the polymials are:

$f(x) = 2x^3 + ax^2 + 3x - 5$

$p(x) = x^3 + x^2 - 4x + a$

The remainders are f(2) and p(2) when f(x) and p(x) are divided by x - 2

We know that,

f(2) = p(2) (given in problem)

we need to calculate f(2) and p(2)

for, f(2)

substitute (x = 2) in f(x)

$f(2) = 2(2)^3 + a(2)^2 + 3(2) - 5$

$= (2 \times 8) + 4a + 6 - 5$

= 16 + 4a + 1

= 4a + 17 ...(1)

for, p(2)

substitute (x = 2) in p(x)

$p(2) = 2^3 + 2^2 - 4(2) + a$

= 8 + 4 - 8 + a

= 4 + a ...(2)

Since, f(2) = p(2)

Equate equation 1 and 2

$\Rightarrow$ 4a + 17 = 4 + a

$\Rightarrow$ 4a - a = 4 - 17

$\Rightarrow$ 3a = -13

$\Rightarrow \ a = \frac{-13}{3}$

The value of $a = \frac{-13}{3}$.

132. In the following, using the remainder theorem, find the remainder when f(x) is divided by g(x) and verify the by actual division:

$f(x) = 4x^4 - 3x^3 - 2x^2 + x - 7$, g(x) = x - 1

Ans. : Here,

$f(x) = 4x^4 - 3x^3 - 2x^2 + x - 7$

$g(x) = x - 1$

From, the remainder theorem when f(x) is divided by g(x) = x - (-1) the remainder will be equal to f(1)

Let, g(x) = 0

⇒ x - 1 = 0

⇒ x = 1

Substitute the value of x in f(x)

$f(1) = 4(1)^4 - 3(1)^3 - 2(1)^2 + 1 - 7$

= 4 - 3 - 2 + 1 - 7

= 5 - 12

= -7

Therefore, the remainder is 7.

133. If x = 0 and x = -1 are the roots of the polynomial $f(x) = 2x^3 - 3x^2 + ax + b$, Find the of a and b.

Ans. : We know that, $f(x) = 2x^3 - 3x^2 + ax + b$

Given, the values of x are 0 and -1

Substitute x = 0 in f(x)

$f(0) = 2(0)^3 - 3(0)^2 + a(0) + b$

= 0 - 0 + 0 + b

= b ...(1)

Substitute x = (-1) in f(x)

$f(-1) = 2(-1)^3 - 3(-1)^2 + a(-1) + b$

= -2 - 3 - a + b

= -5 - a + b ...(2)

We need to equate equations 1 and 2 to zero

b = 0 and -5 - a + b = 0

since, the value of b is zero

substitute b = 0 in equation 2

⇒ -5 - a = -b

⇒ -5 - a = 0

a = -5

the values of a and b are -5 and 0 respectively.

134. Evaluate:

$(28)^3 + (-15)^3 + (-13)^3$

Ans. : $(28)^3 + (-15)^3 + (-13)^3$

We know:

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$x^3 + y^3 + z^3 = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) + 3xyz$

Here, x = (-28), y = -15, z = -13

$(28)^3 + (-15)^3 + (-13)^3$

$= (28 - 15 - 13)[(28)^2 + (-15)^2 + (-13)^2 - 28(-15) - (-15)(-13) - 28(-13)] + 3 \times 28(-15)(-13)$

$= 0 + 16380$

$= 16380$

135. Factorise:

$a^3(b - c)^3 + b^3(c - a)^3 + c^3(a - b)^3$

Ans. : We have:

$a^3(b - c)^3 + b^3(c - a)^3 + c^3(a - b)^3 =$

$[a(b - c)]^3 + [a(b - c)]^3 + [b(c - a)]^3 + [c(a - b)]^3$

Put,

$a(b - c) = x, b(c - a) = y, c(a - b) = z$

Here,

$x + y + z = a(b - c) + b(c - a) + c(a - b)$

$= ab - ac + bc - ab - ab + ac - bc$

Thus,

We have:

$a^3(b - c)^3 + b^3(c - a)^3 + c^3(a - b)^3 = x^3 + y^3 + z^3$

$= 3xyz$ [When $x + y + z = 0, x^3 + y^3 + z^3 = 3xyz$]

$= 3a(b - c)b(c - a)c(a -b)$

$= 3abc(a - b)(b - c)(c - a)$

* Answer the following questions. [5 Marks Each] [65]

136. If $x + \frac{1}{x} = 3$, then find the value of $x^6 + \frac{1}{x^6}$.

Ans. : We have to find the value of $x^6 + \frac{1}{x^6}$

Given $x + \frac{1}{x} = 3$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

Here $a = x,\ b = \frac{1}{x}$

$\left(x + \frac{1}{x}\right)^2 = x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2$

$\left(x + \frac{1}{x}\right)^2 = x^2 + 2 \times \cancel{x} \times \frac{1}{\cancel{x}} + \frac{1}{x} \times \frac{1}{x}$

$\left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}$

By substituting the value of $x + \frac{1}{x} = 3$ We get,

$(3)^2 = x^2 + 2 + \frac{1}{x^2}$

$3 \times 3 = x^2 + 2 + \frac{1}{x}^2$

By transposing + 2 to left hand side, we get

$9 - 2 = x^2 + \frac{1}{x^2}$

$7 = x^2 + \frac{1}{x^2}$

Cubing on both sides we get,

$(7)^3 = x^2 + \left(\frac{1}{x^2}\right)^3$

Using identity $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Here $a = x^2,\ b = \frac{1}{x^2}$

$343 = (x^2)^3 + \left(\frac{1}{x^2}\right)^3 + 3 \times x^2 \times \frac{1}{x^2}\left(x^2 + \frac{1}{x^2}\right)$

$343 = x^6 + \frac{1}{x^6} + 3 \times x^2 \times \frac{1}{x^2}\left(x^2 + \frac{1}{x^2}\right)$

Put $x^2 + \frac{1}{x^2} = 7$ we get

$343 = x^6 + \frac{1}{x^6} + 3 \times 7$

$343 = x^6 + \frac{1}{x^6} + 21$

By transposing 21 to left hand side we get ,

$343 - 21 = x^6 + \frac{1}{x^6}$

$322 = x^6 + \frac{1}{x^6}$

Hence the value of $x^6 + \frac{1}{x^6}$ is 322.

137. If $x + \frac{1}{x} = 3$, calculate $x^2 + \frac{1}{x^2},\ x^3 + \frac{1}{x^3}$ and $x^4 + \frac{1}{x^4}$.

Ans. : In the given problem, we have to find the value of $x^2 + \frac{1}{x^2},\ x^3 + \frac{1}{x^3},\ x^4 + \frac{1}{x^4}$

Given $x + \frac{1}{x} = 3,$

We shall use the identity $(x + y)^2 = x^2 + y^2 + 2xy$

Here putting $x + \frac{1}{x} = 3,$

$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x}$

$(3)^2 = x^2 + \frac{1}{x^2} + 2 \times \cancel{x} \times \frac{1}{\cancel{x}}$

$9 = x^2 + \frac{1}{x^2} + 2$

$9 - 2 = x^2 + \frac{1}{x^2}$

$7 = x^2 + \frac{1}{x^2}$

Again squaring on both sides we get,

$\left(x^2 + \frac{1}{x^2}\right)^2 = (7)^2$

We shall use the identity $(x + y)^2 = x^2 + y^2 + 2xy$

$\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2}$

$(7)^2 = x^4 + \frac{1}{x^4} + 2 \times \cancel{x^2} \times \frac{1}{\cancel{x^2}}$

$49 = x^4 + \frac{1}{x^4} + 2$

$49 - 2 = x^4 + \frac{1}{x^4}$

$47 = x^4 + \frac{1}{x^4}$

Again cubing on both sides we get,

$\left(x+\frac{1}{x}\right)^3=(3)^3$

We shall use the identity (a + b)^3 = a^3 + b^3 + 2ab

$\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)$

$(3)^3=x^3+\frac{1}{x^3}+3\times\cancel{x}\times\frac{1}{\cancel{x}}\times 3$

$27=x^3+\frac{1}{x^3}+9$

$27-9=x^3+\frac{1}{x^3}$

$18=x^3+\frac{1}{x^3}$

Hence the value of $x^2+\frac{1}{x^2},\ x^3+\frac{1}{x^3},\ x^4+\frac{1}{x^4}$ is 7, 18, 47 respectively.

138. If $x^4+\frac{1}{x^4}=119$, find the valu of $x^3-\frac{1}{x^3}$.

Ans. : In the given problem, we have to find the value of $x^3-\frac{1}{x^3}$

Given $x^4+\frac{1}{x^4}=119$

We shall use the idntity (x + y)^2 = x^2 + y^2 + 2xy

Here putting $x^4+\frac{1}{x^4}=119$,

$\left(x^2+\frac{1}{x^2}\right)^2=x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2}$

$\left(x^2+\frac{1}{x^2}\right)^2=x^4+\frac{1}{x^4}+2\times\cancel{x^2}\times\frac{1}{\cancel{x^2}}$

$\left(x^2+\frac{1}{x^2}\right)^2=x^4+\frac{1}{x^4}+2$

$\left(x^2+\frac{1}{x^2}\right)^2=119+2$

$\left(x^2+\frac{1}{x^2}\right)^2=121$

$x^2+\frac{1}{x^2}=\sqrt{11\times 11}$

$x^2+\frac{1}{x^2}=\pm 11$

In order to find $\left(x-\frac{1}{x}\right)$ we are using identity (x - y)^2 = x^2 + y^2 - 2xy

$\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2\times x\times\frac{1}{x}$

$\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2$

$\left(x-\frac{1}{x}\right)^2=11-2$

$\left(x-\frac{1}{x}\right)^2=9$

$\left(x-\frac{1}{x}\right)=\sqrt{9}$

$\left(x-\frac{1}{x}\right)=\sqrt{3\times 3}$

$\left(x - \frac{1}{x}\right) = \pm 3$

In order to find $\left(x^3 - \frac{1}{x^3}\right)$ we are using identity $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + x \times \frac{1}{x}\right)$

Here $x^2 + \frac{1}{x^2} = 11$ and $\left(x - \frac{1}{x}\right) = 3$

$x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + \cancel{x} \times \frac{1}{\cancel{x}}\right)$

$= 3(11 + 1)$

$= 3 \times 12$

$= 36$

Hence the value of $x^3 - \frac{1}{x^3}$ is 36.

139. Simplify the following expressions:

$\left(x + y + z\right)^2 + \left(x + \frac{y}{2} + \frac{z}{3}\right)^2 - \left(\frac{x}{2} + \frac{y}{3} + \frac{z}{4}\right)^2$

Ans. : Expanding, we get

$zx = \left[x^2 + y^2 + z^2 + 2xy + 2yz + 2\right]$

$+ \left[x^2 + \frac{y^2}{4} + \frac{z^2}{9} + 2x\frac{y}{2} + 2\frac{zx}{3} + \frac{yz}{3}\right]$

$- \left[\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{10} + \frac{xy}{3} + \frac{yz}{6} + \frac{xz}{4}\right]$

$\left[\therefore \left(x + y + z\right)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\right]$

$= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx + x^2 + \frac{y^2}{4} + \frac{z^2}{9}$

$+ 2x\frac{y}{2} + \frac{xy}{3} + \frac{2zx}{3} - \frac{x^2}{4} - \frac{y^2}{9} - \frac{z^2}{10} - \frac{xy}{3} - \frac{yz}{6} - \frac{xz}{4}$

Rearranging coefficients,

$= \frac{8x^2 - x^2}{4} + \frac{36y^2 + 9y^2 - 4y^2}{36} + \frac{144z^2 + 16z^2 - 9z^2}{144}$

$+ \frac{6xy + 3xy - xy}{3} \frac{13yz}{6} + \frac{29xz}{12}$

$= \frac{7x^2}{4} + \frac{41y^2}{36} + \frac{151z^2}{144} + \frac{8xy}{3} + \frac{13yz}{6} + \frac{29zx}{12}$

$\left(x + y + z\right)^2 + \left(x + \frac{y}{3} + \frac{z}{3}\right)^2 - \left(\frac{x}{2} + \frac{y}{3} + \frac{z}{4}\right)^2$

$= \frac{7x^2}{4} + \frac{41y^2}{36} + \frac{151z^2}{144} + \frac{8xy}{3} + \frac{13yz}{6} + \frac{29zx}{12}$

140. Simplify the following:

$\left(x + \frac{2}{x}\right)^3 + \left(x - \frac{2}{x}\right)^3$

Ans. : Given $\left(x + \frac{2}{x}\right)^3 + \left(x - \frac{2}{x}\right)^3$

We shall use the identity $a^3 + b^3 = (a - b)(a^2 + b^2 - ab)$

Here $a = \left(x + \frac{2}{x}\right),\ b = \left(x - \frac{2}{x}\right)$

By applying identity we get

$$=\left(x+\frac{2}{x}+x-\frac{2}{x}\right)\left[\left(x+\frac{2}{x}\right)^2+\left(x-\frac{2}{x}\right)^2\right.$$

$$\left.-\left(\left(x+\frac{2}{x}\right)\times\left(x-\frac{2}{x}\right)\right)\right]$$

$$=\left(x+\frac{2}{x}+x-\frac{2}{x}\right)\left[\left(x\times x+\frac{2}{x}\times\frac{2}{x}+2\times x\times\frac{2}{x}\right)\right.$$

$$\left.+\left(x\times x+\frac{2}{x}\times\frac{2}{x}-2\times x\times\frac{2}{x}\right)-\left(x^2+\frac{4}{x^2}\right)\right]$$

$$=(2x)\left[\left(x^2+\frac{4}{x^2}+\frac{4x}{x}\right)+\left(x^2+\frac{4}{x^2}-\frac{4x}{x}\right)-\left(x^2-\frac{4}{x^2}\right)\right]$$

$$=(2x)\left[x^2+\frac{4}{x^2}+\frac{4x}{x}+x^2+\frac{4}{x^2}-\frac{4x}{x}-x^2+\frac{4}{x^2}\right]$$

By rearranging the variable we get,

$$=(2x)\left[x^2+\frac{4}{x^2}+\frac{4}{x^2}+\frac{4}{x}^2\right]$$

$$=2x\times\left[x^2+\frac{12}{x^2}\right]$$

$$=2x^2+\frac{24}{x}$$

Hence the simplified value of $\left(x+\frac{2}{x}\right)^3+\left(x-\frac{2}{x}\right)^3$ is $2x^2+\frac{24}{x}$.

141. If 3x = a + b + c, then the value of $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c)$ is:

a. a + b + c
b. (a - b)(b - c)(c - a)
c. 0
d. None of these.

Ans. :

c. 0

Solution:

3x = a + b + c

⇒ a + b + c - 3x = 0

⇒ 3x - (a + b + c) = 0

⇒ (x - a) + (x - b) + (x - c) = 0 ...(1)

Using identity if a + b + c = 0 then, $a^3 + b^3 + c^3 - 3abc = 0$

If we take x - a = A, x - b = B, x - c = C in equation (1), we get

A + B + C = 0

⇒ $A^3 + B^3 + C^3 - 3ABC = 0$

⇒ $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c) = 0$

Hence, correct option is (c).

142. If $x^3 - 3x^2 + 3x - 7 = (x + 1)(ax^2 + bx + c)$, then a + b + c =

a. 4

b. 12
c. -10
d. 3

Ans. :

c. -10

Solution:

The given equation is

$x^3 - 3x^2 + 3x - 7 = (x + 1)(ax^2 + bx + c)$

This can be written as

$x^3 - 3x^2 + 3x - 7 = (x + 1)(ax^2 + bx + c)$

$= x^3 - 3x^2 + 3x - 7 = ax^3 + bx^2 + cx + ax^2 + bx + c$

$= x^3 - 3x^2 + 3x - 7 = ax^3 + (a + b)x^2 + (b + c)x + c$

Comparing the cofficients on both sides of the equation.

We get,

$a = 1$ ...(1)

$a + b = 3$ ...(2)

$b + c = 3$ ...(3)

$c = -7$ ...(4)

Putting the value of a form (1) in (2)

We get,

$1 + b = 3,$

$b = -3 - 1$

$b= -4$

So the value of a, b and c is 1, -4 and -7 respectively.

Therefore,

$a + b + c = 1 - 4 - 7 = -10$

Hence, correct option is (c).

143. If $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^2$, then k =

a. 1
b. 2
c. 4
d. 8

Ans. :

d. 8

Solution:

Let $x + y = A$ and $x - y = B$

Now, $(A - B)^3 = A^3 - B^3 - 3AB(A - B)$

$\Rightarrow [(x + y) - (x - y)]^3 = (x + y)^3 - (x - y)^3 - 3(x + y)(x - y)[(x + y) - (x - y)]$

$= (x + y)^3 - (x - y)^3 - 3(x^2 - y^2)(2y)$

$= (x + y)^3 - (x - y)^3 - 6y(x^2 - y^2)$

But, $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^3$

$\Rightarrow [(x + y) - (x - y)]^3 = (2y)^3 = k8y^3$

$\Rightarrow (2y)^3 = ky^3$

$\Rightarrow 8y^3 = ky^3$

$\Rightarrow k = 8$

Hence, correct option is (d).

144. If 2 and 0 are the zeros of the polynomial $f(x) = 2x^3 - 5x^2 + ax + b$ then find the values of a and b.

Hint: f(x) = 0 and f(0) = 0.

Ans. : It is given that 2 and 0 are the zeros of the polynomial $f(x) = 2x^3 - 5x^2 + ax + b$.

$\therefore f(2) = 0$

$\Rightarrow 2 \times 2^3 - 5 \times 2^2 + a \times 2 + b = 0$

$\Rightarrow 16 - 20 + 2a + b = 0$

$\Rightarrow -4 + 2a + b = 0$

$\Rightarrow 2a + b = 4$ ...(1)

Also,

$f(0) = 0$

$\Rightarrow 2 \times 0^3 - 5 \times 0^2 + a \times 0 + b = 0$

$\Rightarrow 0 - 0 + 0 + b = 0$

$\Rightarrow b = 0$

Putting b = 0 in (1), we get

$2a + 0 = 4$

$\Rightarrow 2a = 4$

$\Rightarrow a = 2$

Thus, the values of a and b are 2 and 0, respectively.

145. The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + b$ when divided by (x - 1) and (x + 1) leaves the remainders 5 and 19 respectively. Find the values of a and b. Hence, find the remainder when p(x) is divided by (x - 2).

Ans. : Let:

$p(x) = x^4 - 2x^3 + 3x^2 - ax + b$

Now,

When p(x) is divided by (x - 1), the remainder is p(1).

When p(x) is divided by (x + 1), the remainder is p(-1).

Thus, we have:

$p(1) = (1^4 - 2 \times 1^3 \times + 3 \times 1^2 - a \times 1 + b)$

$= (1 - 2 + 3 - a + b)$

$= 2 - a + b$

And,

$p(-1) = [(-1)^4 - 2 \times (-1)^3 + 3 \times (-1)^2 - a \times (-1) + b]$

$= (1 + 2 + 3 + a + b)$

$= 6 + a + b$

Now,

$2 - a + b = 5$ ...(1)

$6 + a + b = 19$ ...(2)

Adding (1) and (2), we get:

$8 + 2b = 24$

$\Rightarrow 2b = 18$

$\Rightarrow b = 8$

By putting the value of b, we get the value of a, i.e., 5.

$\therefore a = 5$ and $b = 8$

Now,

$f(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$

Also,

When p(x) is divided by (x - 2), the remainder is p(2).

thus, we have:

$p(2) = (2^4 - 2 \times 2^3 + 3 \times 2^2 - 5 \times 2 + 8)$ [a = 5 and b = 8]

$= (16 - 16 + 12 - 10 + 8)$

$= 10$

146. Find the values of a and b so that the polynomial $(x^4 + ax^3 - 7x^2 - 8x + b)$ is exactly divisible by $(x + 2)$ as well as $(x + 3)$.

Ans. : Let $f(x) = (x^4 + ax^3 - 7x^2 - 8x + b)$

Now, $x + 2 = 0$

$\Rightarrow x = -2$ and,

$\Rightarrow x + 3 = 0$

$\Rightarrow x = -3$

By factor theorem, $(x + 2)$ and $(x + 3)$ will be factors of f(x) if $f(-2) = 0$ and $f(-3) = 0$

$\therefore f(-2) = (-2)^4 + a(-2)^3 - 7(-2)^2 - 8(-2) + b = 0$

$\Rightarrow 16 - 8a - 28 + 16 + b = 0$

$\Rightarrow -8a + b = -4$

$\Rightarrow 8a - b = 4$ ...(i)

And, $f(-3) = (-3)^4 + a(-3)^3 - 7(-3)^2 - 8(-3) + b = 0$

$\Rightarrow 81 - 27a - 63 + 24 + b = 0$

$\Rightarrow -27a + b = -42$

$\Rightarrow 27a - b = 42$ ...(ii)

Subtracting (i) from (ii), we get,

$19a = 38$

So, $a = 2$

Substituting the value of $a = 2$ in (i), we get

$8 \times 2 - b = 4$

$\Rightarrow 16 - b = 4$

$\Rightarrow -b = -16 + 4$

$\Rightarrow -b = -12$

$\Rightarrow b = 12$

$\therefore a = 2$ and $b = 12$.

147. If $(x^3 + ax^2 + bx + 6)$ has $(x - 2)$ as a factor and leaves a remainder 3 when divided by $(x - 3)$, find the values of a and b.

Ans. : Let $f(x) = (x^3 + ax^2 + bx + 6)$

Now, by remainder theorem, f(x) when divided by (x - 3) will leave a remainder as f(3).

$\Rightarrow$ So, $f(3) = 3^3 + a\ 3^2 + b\ 3 + 6 = 3$

$\Rightarrow 27 + 9a + 3b + 6 = 3$

$\Rightarrow 9a + 3b + 33 = 3$

$\Rightarrow 9a + 3b = 3 - 33$

$\Rightarrow 9a + 3b = -30$

$\Rightarrow 3a + b = -10$ ...(i)

Given that (x - 2) is a factor of f(x).

By the Factor Theorem, (x - a) will be a factor of f(x) if f(a) = 0 and therefore f(2) = 0.

$\Rightarrow f(2) = 2^3 + a\ 2^2 + b\ 2 + 6 = 0$

$\Rightarrow 8 + 4a + 2b + 6 = 0$

$\Rightarrow 4a + 2b = -14$

$\Rightarrow 2a + b = -7$ ...(ii)

Subtracting (ii) from (i), we get,

$\Rightarrow a = -3$

Substituting the value of a = -3 in (i), we get,

$3(-3) + b = -10$

$\Rightarrow -9 + b = -10$

$\Rightarrow b = -10 + 9$

$\Rightarrow b = -1$

$\therefore a = -3$ and $b = -1$.

148. What must be subtracted from $(x^4 + 2x^3 - 2x^2 + 4x + 6)$ so that the result is exactly divisible by $(x^2 + 2x - 3)$?

Ans. : Let $p(x) = x^4 + 2x^3 - 2x^2 + 4x + 6$ and $q(x) = x^2 + 2x - 3$.

When p(x) is divided by q(x), the remainder is a linear expression in x.

So, let r(x) = ax + b be subtracted from p(x) so that p(x) - r(x) is divided by q(x).

Let f(x) = p(x) - r(x) = p(x) - (ax + b)

$= (x^4 + 2x^3 - 2x^2 + 4x + 6) - (ax + b)$

$= x^4 + 2x^3 - 2x^2 + (4 - a)x + 6 - b$

We have,

$q(x) = x^2 + 2x - 3$

$= x^2 + 3x - x - 3$

$= x(x + 3) - 1(x + 3)$

$= (x + 3)(x - 1)$

Clearly, (x + 3) and (x - 1) are factors of q(x).

Therefore, f(x) will be divisible by q(x) if (x + 3) and (x - 1) are factors of f(x).
i.e., f(-3) = 0 and f(1) = 0
Consider, f(-3) = 0
⇒ $(-3)^4 + 2(-3)^3 - 2(-3)^2 + (4 - a)(-3) + 6 - b = 0$
⇒ 81 - 54 - 18 - 12 + 3a + 6 - b = 0
⇒ 3 + 3a - b = 0
⇒ 3a - b = -3 ...(i)
And, f(1) = 0
⇒ $(1)^4 + 2(1)^3 - 2(1)^2 + (4 - a)(1) + 6 - b = 0$
⇒ 1 + 2 - 2 + 4 - a + 6 - b = 0
⇒ 11 - a - b = 0
⇒ -a - b = -11 ...(ii)
Subtracting (ii) from (i), we get
4a = 8
⇒ a = 2
Substituting a = 2 in (i), we get
3(2) - b = -3
⇒ 6 - b = -3
⇒ b = 9
Putting the values of a and b in r(x) = ax + b, we get
r(x) = 2x + 9
Hence, p(x) is divisible by q(x), if r(x) = 2x + 9 is subtracted from it.

*** Case study based questions. [8]**

149. Hard plastic square shaped sheets are available in the.
The side length of sheets is as per requirement.
The price of a sheet is z per square meter.
Anuj requires two sheets – a smaller sheet with side length x m and a larger sheet with side length ym. He has two choices:
Choice 1 – buy two separate sheets of side lengths x m and y m
Choice 2 – buy a single sheet with side length (x+ y) m
4. What is the height of each container?
5. What is the difference in price between the two choices?
6. The area of a rectangle is $(3x^2 + x - 2)$ square units. Its width is $(1 + x)$ units. What is the length of the rectangle?
7. A polynomial is expressed as $x^3 + bx^2 + cx + d = 0$. The same polynomial can be written in factor form as $x + px + qx + r = 0$.
How is the constant term in the polynomial related to its factors p, q, and r?
A. $d = p + q + r$
B. $d = (p + q) \times r$
C. $d = p \times q \times r$
D. $d = pq + qr + pr$
8. A polynomial is divided by $(x - 1)$. The quotient obtained is $3x^3 - x^2 - x - 4$, and

the remainder is -5 . Which polynomial meets these conditions?

A. $3x^3 - x^2 - x - 9$

B. $3x^3 - x^2 - x - 4$

C. $3x^4 - 4x^3 - 3x + 4$

D. $3x^4 - 4x^2 - 3x - 1$

9. What is the common factor of $x^3 - x^2$ and $-22x^2 + 142x - 120$?

A. x

B. (x – 1)

C. x^2

D. 1

10. A polynomial is expressed as: $p(x) = x^3 + x^2 - x - 1$

At what values of x is the polynomial $p(x) = 0$?

Ans. : 4. Mentions Choice 1 OR 1

5. Writes 2 x y z with or without the word 'units'

● 2 x y z

● 2 x y z units

6. Writes 3x – 2 with or without the word 'units'

● 3x – 2 units

● 3x – 2

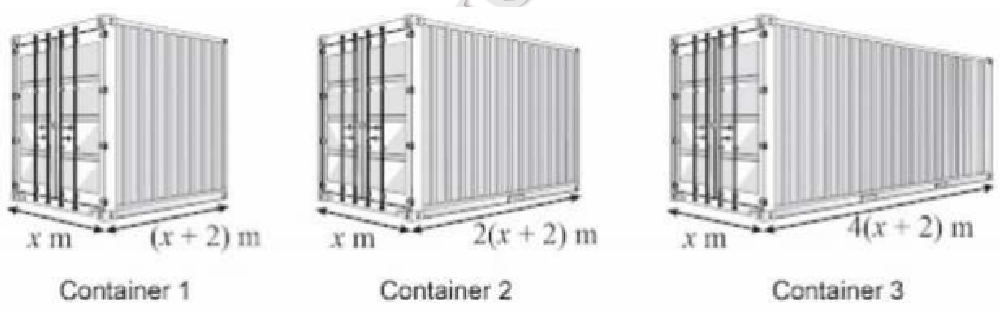
7. C. d = p × q × r

8. D. $3x^4 - 4x^3 - 3x - 1$

9. B. (x – 1)

10. Writes 1 and –1

150. A shipment service provider uses three types of containers for shipping materials. The height and
width of the three containers are the same. The containers' height is 0.15 m more than their width, and the volume of the smallest container is 652 m^3



1. Write a polynomial relating Container 1's length, breadth and height with its volume.

2. Which of the following statements is true?

A. The volume of the three containers is the same.

B. The length of the three containers is the same.

C. The volume of Container 3 is 2,608 m^3.

D. The length of Container 3 is 4 times the length of Container 2.

3. What is the height of each container?

Ans. : 1. Writes an equation relating length, breadth, height and volume.
- $x^3 + 2.15x^2 + 0.3x = 652$
- $x^3 + 2.15x^2 + 0.3x - 652 = 0$
- $x(x+2)(x+0.15) = 652$
- $x(x+2)(x+0.15) - 652 = 0$

2. C. The volume of Container 3 is 2608 m^3

3. Write 8.15 with or without the Chapter

- 8.15 m
- 8.15

----- "अगर किसी चीज को शिद्दत से चाहो तो पूरी कायनात तुम्हें उससे मिलाने में लग जाती है", -----